Outlier Robust Unit Root Tests in Nonlinear Dynamic Models

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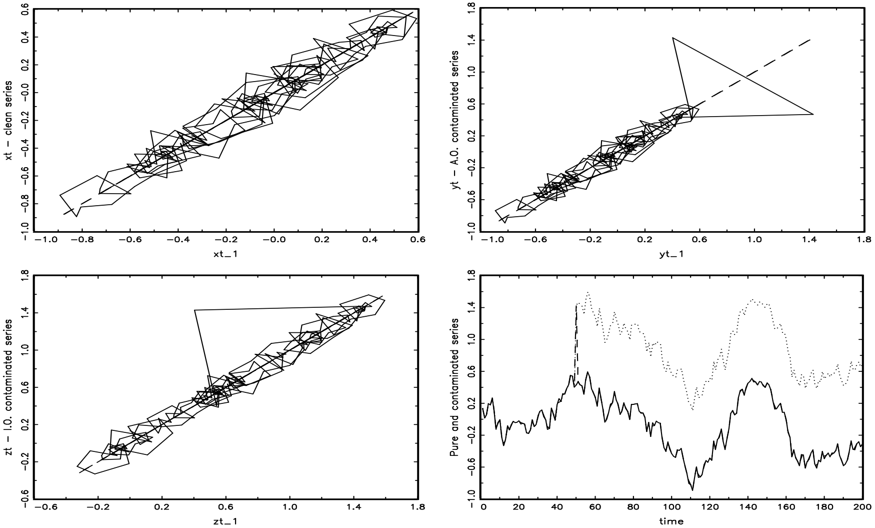
Introduction

- Testing the unit root hypothesis is an important step in the analysis of economic time series
- However, the findings of traditional unit root tests (LS based) may be spurious in the case of outliers (Franses and Haldrup, 1994; Sandberg, 2015, 2016)
- ...and there are substantial evidence of outliers in data (Balke and Fomby, 1994)
- Two common approaches to remedy the problem of outliers: (i) test for them, and remove them if needed (ii) robust estimation
- Two types of outliers: Innovation outliers (IOs) - the onset of an external cause (e.g., financial crises); Additive Outliers (AOs) - recording or measurement errors

Figure 1 Scatter plots for IO and AO unit root series



- In general, outliers may cause undesirable effects in terms of estimation (finite sample bias); limiting distributions may be shifted causing size distorted tests; and the power of the tests may be adversely affected
- In fact, the LS-estimator has BdP equal to 0. The estimator used in this work has BdP ranging from 0 to .25
- The work on outlier robust unit root tests in nonlinear models is scarce
- Of course, outliers are not only a problem in a unit root context; there is a large body of work on outliers in the context of linear and nonlinear stationary models

- The contribution of my work lies in the that M-estimator based unit root tests in general nonlinear dynamic models are provided
- As such, my main focus is to derive unit root tests that are robust against IOs
- In my nonlinear framework, IOs have a permanent effect under the null hypothesis and yield a unit root process with level shift(s)
- Under the alternative hypothesis, IOs have a temporary effect due to the (presumed) ergodicity properties of the nonlinear models...
- ...but the regime switching behavior is affected. For instance, IOs can cause (i) "additional" regime shifts (from recession to boom, say) (ii) a recession (boom) to be even more pronounced - this is further illustrated in the application

The Nonlinear Models, their Approximations and Unit Roots

- Consider a stochastic process $(Y_t)_{t \geq 1}$ generated by the first-order (possibly) nonlinear dynamic model (a STR-type model; Teräsvirta et al, 2010):

$$Y_t = \pi_{10} + \pi_{11} Y_{t-1} + [\pi_{20} + \pi_{21} Y_{t-1}] G(Z_t) + u_t \qquad (1)$$

  where Y_0 is a "well-behaved" starting value, (π_{i0}, π_{i1}) $(i = 1, 2)$ are real-valued parameters, $G(Z_t)$ is a (possibly) nonlinear function and Z_t is the transition variable, and u_t is a strong-mixing error term

- Under some conditions, $G(Z_t)$ can be expressed, using a Taylor-series approximation around $Z_t = 0$, as

$$G(Z_t) = \sum_{n=1}^{k} \frac{\nabla^n G(0)}{n!} Z_t^n + R(Z_t) \tag{2}$$

- Substituting for the approximation (2) into (1), letting $Z_t = Y_{t-1}$, yields the regression equation

$$Y_t = \tilde{X}_t' \beta + e_t \tag{3}$$

where $\tilde{X}_t = [1, Y_{t-1}, Y_{t-1}^2, ..., Y_{t-1}^{k+1}]'$, $\beta = [\beta_0, \beta_1, ..., \beta_{k+1}]'$, and $e_t = R(Z_t) + u_t$

- The unit root hypothesis in (3) is tested by

$$\tilde{\mathrm{H}}_0 : \beta_1 = 1 \text{ and } \beta_m = 0 \text{ for } m \neq 1 \tag{4}$$

under which (3) reduces to the random walk: $Y_t = Y_{t-1} + u_t$

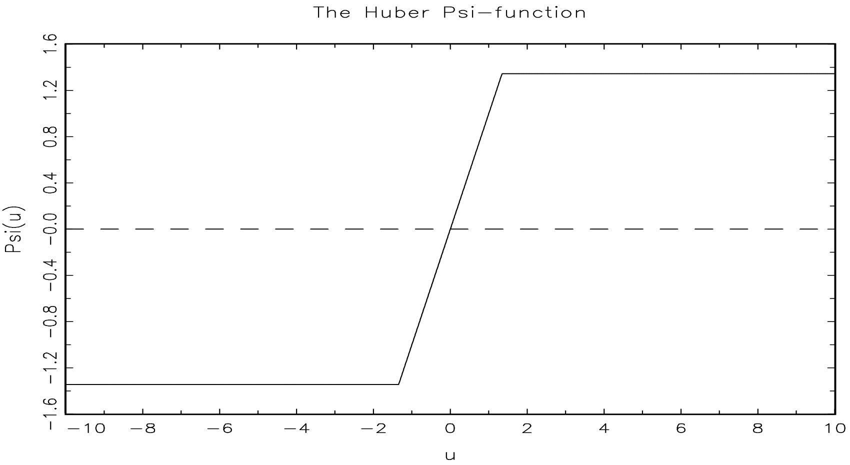
The LS- and M-Estimators

- The LS-estimator $\hat{\beta}_{LS}$ of β in (3) will be used as a benchmark estimator
- The M-estimator for β in (3) is defined as a real-valued vector $\hat{\beta}_{\psi}$ which solves the first-order condition

$$\sum_t \psi(\hat{e}_t)\tilde{X}_t = 0_{(k+1)\times 1}$$

  where $\psi(\cdot)$ is a real-valued function satisfying some regularity conditions, and $\hat{e}_t = Y_t - \tilde{X}_t'\hat{\beta}_{\psi}$
- It is noted that the LS-estimator is obtained as a special case letting $\psi(\hat{e}_t) = \hat{e}_t$

- In this work, the Huber influence function is used: $\psi(u_t) = \min\{c, \max(-c, u_t)\}$, with $c = 1.345$



Unit Root Testing and (some) Large Sample Results

- The unit root hypothesis in (4) is tested by the Wald test statistic:

$$W_{\psi}(k) \triangleq (\hat{\beta}_{\psi} - \beta)' V_{\psi,T}^{-1} \left(\hat{\beta}_{\psi} - \beta\right)$$

- As a benchmark test I also consider:

$$W_{LSH}(k) \triangleq (\hat{\beta}_{LS} - \beta)' V_{W,T}^{-1} (\hat{\beta}_{LS} - \beta)$$

Theorem (Limiting Distributions Free of Nuisance Parameters)

Under some assumptions and regularity conditions (stated in the paper),

$$\tilde{W}_{\psi}(k) \Rightarrow \left(\int_0^1 \mathcal{B} db_2\right)' \left(\int_0^1 \mathcal{B}\mathcal{B}'\right)^{-1} \left(\int_0^1 \mathcal{B} db_2\right)$$

$$\tilde{W}_{LSH}(k) \Rightarrow \left(\int_0^1 \mathcal{B} db_1\right)' \left(\int_0^1 \mathcal{B}\mathcal{B}'\right)^{-1} \left(\int_0^1 \mathcal{B} db_1\right)$$

for $k \in \mathbb{Z}_+$, and where $\mathcal{B} = \left(1, b_1, b_1^2, ..., b_1^{k+1}\right)'$, and b_1 and b_2 are two dependent Brownian motions

- The limiting distribution for $\tilde{W}_{\psi}$ depends on k (the order of the approximation) and ψ. The limiting distribution for $\tilde{W}_{LSH}$ depends on k only
- Interestingly, letting $k = 0$ (a linear model), the results by Lucas (1995) and Phillips (1987) are (about) obtained

Size and Power Studies

- In general, IOs do not cause the tests to be size-distorted (it is another story for AOs, though)
- In general, considering nonlinear alternatives with IOs, the robust tests yield significant power gains over the LS based ones
- Considering nonlinear alternatives with *no* IOs, the robust tests are relatively efficient in terms of power as compared to the LS based ones

Application

- The unit root hypothesis is examined for the eight real effective exchange rate series (REER): Australia (AU), Canada (CA), France (FR), Germany (GE), Japan (GE), Netherlands (NE), United Kingdom (UK) and United States of America (US)
- The sample is based on quarterly data ranging from 1980Q1 to 2012Q2 ($T = 130$)
- The tests are based on $\tilde{X}_t = [1, Y_{t-1}, Y_{t-1}^2, Y_{t-1}^3]'$ (i.e., $k = 2$) having the following three-regime STAR(1) model in mind:

$$Y_t = \pi_{10} + \pi_{11} Y_{t-1} + [\pi_{20} + \pi_{21} Y_{t-1}] G(Y_{t-1}) + u_t$$

with

$$G(Y_{t-1}) = 1/[1 + \exp\{-\gamma(Y_{t-1} - c_1)(Y_{t-1} - c_2)\}]$$

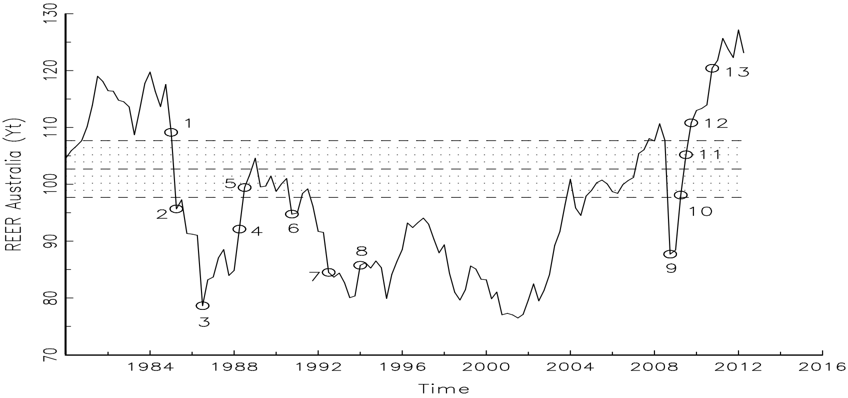
Table Testing the unit root hypothesis in REER series

	AU	CA	FR	GE	JA	NE	UK	US
$\tilde{W}_{\psi}(2)$	**	-	**	*	*	**	*	-
$\tilde{W}_{LSH}(2)$	*	-	*	*	*	**	*	-
$\tilde{W}_{\psi}(0)$	-	-	*	-	-	*	-	-
$\tilde{W}_{LSH}(0)$	-	-	**	-	-	*	-	-
t_{KSS}	-	-	-	-	*	**	*	-
t_{PP}	-	-	*	-	-	*	-	-

Notes: * and ** signify rejection at the 10% and 5% level, respectively. - signifies no rejection.

- The unit root hypothesis is rejected for all countries except Canada and US
- Some estimation results for the AU REER series can be summarized as follow

Figure 2 The Australian REER series



- Circles indicate outliers detected by the Tsay's IO test (at a 5% significance level) using a linear filter

Concluding Remarks

- Outlier robust unit root tests in first-order STR models with strong mixing innovations are derived
- The nonlinearities can be set quite general as long as they admit a Taylor-series approximation (kth-order approximations are allowed for)
- Asymptotic results for outlier robust tests in linear models as well as theory for LS-based unit root tests in linear and nonlinear models merge as special cases
- The size properties of the outlier robust tests are satisfactory under IOs, and they are more powerful than the LS based ones against STR alternatives with IOs
- In application to REER series, support for the PPP hypothesis is found in 6 out of 8 series using the robust tests