

# Pricing currency risks\*

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## Abstract

The currency market features a relatively small cross-section and conditional expected returns can be characterized by only a few signals – interest differentials, trend and mean-reversion. We exploit these properties to construct a conditional projection of the stochastic discount factor onto excess returns of individual currencies. Our approach is implementable in real time and prices all currencies and prominent strategies conditionally as well as unconditionally. We document that the fraction of unpriced risk in these assets is at least 85%. Extant explanations of carry strategies based on intermediary capital or global volatility are related to these unpriced components, while consumption growth is related to the priced component of returns.

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# 1 Introduction

In this paper we argue that research on cross-sectional currency pricing can depart fruitfully from the factor mining path established by the equity literature. That is because direct *conditional* projection of the stochastic discount factor (SDF), as advocated by [Hansen and Richard \(1987\)](#), is feasible in the exchange-rate setting. The value of this approach, if it is empirically successful, is obvious. One obtains a measure of the SDF that prices, both conditionally and unconditionally, the full cross-section of currencies and trading strategies associated with them. This measure can help with directing research that tries to develop an understanding of currency risk pricing.

Specifically, the projection approach requires two critical ingredients: estimates of the conditional mean and covariance matrix of currency returns. This is where exchange rates have an advantage over equities. First, the size of the cross-section of exchange rates is small, not exceeding 40 as compared to thousands of stocks. That feature dramatically affects the precision of the covariance matrix regardless of the specifics of the estimation method. Second, exchange rates have a very particular economic structure that helps with hypothesizing the functional form of the conditional means. Research that goes back decades suggests the importance of the interest rate differential (Covered Interest Parity, [Keynes, 1923](#); Uncovered Interest Parity, [Porter, 1971](#); Random Walk Hypothesis, [Meese and Rogoff, 1983](#)), mean-reversion (Purchasing Power Parity, [Cassel, 1918](#)), and trend in nominal exchange rate (weak form of market efficiency, [Cornell and Dietrich, 1978](#)). These three drivers remain the key ingredients in the modern currency trading strategies under the names of Carry, Value, and Momentum, respectively.

We work with a sample of G10 currencies, the most commonly used data in the literature, with monthly returns based on forward rates from January 1985 through May 2020. We construct monthly conditional expected excess returns using the three aforementioned signals. The loading on the first signal is one as motivated by the random walk hypothesis, while the loadings on the other signals are estimated on a rolling basis via panel regressions. We construct a conditional covariance matrix of currency returns using daily data within each month. Importantly, the overall approach uses data that are available to investors in real time.

We consider nine leading trading strategies in addition to individual currencies as test assets. The Dollar strategy represents the currency version of “the market” (Lustig, Roussanov, and Verdelhan, 2011). The Dollar Carry goes long or short the Dollar factor depending on the average carry signal (Lustig, Roussanov, and Verdelhan, 2014). Next, we evaluate the cross-sectional and time-series Carry strategies (Burnside, Eichenbaum, Kleshchelski, and Rebelo, 2011, Daniel, Hodrick, and Lu, 2017, Koijen, Moskowitz, Pedersen, and Vrugt, 2018, Lustig, Roussanov, and Verdelhan, 2011). These strategies are followed by two versions of cross-sectional and time-series Momentum depending on whether one uses the past month or year as the measure of past performance (Asness, Moskowitz, and Pedersen, 2013, Burnside, Eichenbaum, and Rebelo, 2011, Burnside, Eichenbaum, Kleshchelski, and Rebelo, 2011, Menkhoff, Sarno, Schmeling, and Schrimpf, 2012b, Moskowitz, Ooi, and Pedersen, 2012). Lastly, we study cross-sectional Value that sorts currencies based on the real exchange rate (Asness, Moskowitz, and Pedersen, 2013). As emphasized by Asness, Moskowitz, and Pedersen (2013) and Koijen, Moskowitz, Pedersen, and Vrugt (2018), besides being the most common strategies in FX, they are more broadly considered in other asset classes.

The estimated SDF can be represented as a linear function of the unconditional mean-variance efficient portfolio (UMVE). The UMVE is tradeable in real time, has a Sharpe ratio (SR) in excess of 1, and, conveniently, allows us to compare directly to existing trading strategies using standard regression-based tests. We verify that it prices both excess returns on nine individual exchange rates vs the U.S. dollar and the nine above-mentioned trading strategies via the unconditional and conditional pricing tests of [Gibbons, Ross, and Shanken \(1989\)](#) and [Chernov, Lochstoer, and Lundebj \(2018\)](#), respectively.

Having tested the SDF, we proceed with evaluating its properties. The UMVE explains only a small fraction of the time-series variation in strategy returns. For instance, less than 1% of the variation in the Dollar factor returns (the currency “market factor”) are priced, while the same for Carry strategies is at most 15%. Thus, there are large unpriced components in strategy returns and we show that hedging out these unpriced components in an out-of-sample manner dramatically increases SR of the hedged strategy returns. As a model-check, we verify that the average returns to the (out-of-sample) unpriced components are indeed not significantly different from zero.

Next, we document that the factors that are the most important for time-series co-variation among currencies and the currency trading strategies have little relation to the UMVE portfolio. In other words, the main factors that drive the covariance matrix of returns are not an important source of risk for the marginal currency investor. For instance, the first two principal components of individual currency returns capture 75% of the return variation and are strongly associated with the Dollar and cross-sectional Carry strategies, consistent with [Verdelhan \(2018\)](#).<sup>1</sup> How-

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<sup>1</sup>[Aloosh and Bekaert \(2019\)](#) and [Greenaway-McGrevy, Mark, Sul, and Wu \(2018\)](#) offer alternative

ever, these principal components explain only around 5% of the variation in UMVE returns. In fact, all nine of these principal components only explain around 10% of the UMVE returns indicating that conditional currency risk premiums vary strongly and underscoring the importance of correct conditioning.

The currency strategies involve a lot of timing and therefore potentially accounts for these conditional dynamics. We investigate whether their principal components are capable of explaining variation in the UMVE. The answer is no, although there is some improvement. The first three principal components of the currency trading strategies explain around 20% of the variation in the UMVE returns. Using all the principal components, this increases to 50%. This evidence is consistent with the uncovered large unpriced component of strategy returns. It also indicates that, despite similar signals used for construction of the UMVE and strategy portfolios, there is no built-in mechanical relationship between them. Taken together, this suggests a large role of optimal timing implicit in the construction of the UMVE above and beyond the timing already implicit in existing trading strategies.

To give a sense of the magnitudes, consider the standard deviation of a strategy's conditional expected returns. The average of this number across strategies, annualized, is 3.4%. For comparison, the average unconditional return across strategies is 3.2%. The large variation in conditional returns suggests a substantial benefit also to individual factor timing. We do so optimally in an out-of-sample fashion by scaling a position in each strategy up or down depending on our estimates of the individual currency conditional expected returns and covariance matrix. In six cases alphas of the timed strategies with respect to their original counterparts are significant with large Information ratios. Thus, there are large gains in terms of the unconditional

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approaches to analysis of time-series behavior of exchange rates.

SR to timing existing factors proposed in the literature.

To interpret the evidence, it is convenient to think of the SDF as consisting of two components. One is the UMVE expressed in terms of a numeraire that is not currency-specific (e.g., a world currency index). The other one reflects the change of numeraire to a given currency. This view is similar to [Lustig, Roussanov, and Verdelhan \(2011\)](#) and [Korsaye, Trojani, and Vedolin \(2020\)](#). We show that, because SR of the UMVE is slightly greater than 1 while the volatility of depreciation rates is about 0.1 (per annum), it must be the case that most of the valuation effect stems from the UMVE component of the SDF. Furthermore, strategy returns are exposed to common sources of variation in currencies that are not related to the UMVE. These common sources are important for explaining the variation in strategy returns (e.g., [Verdelhan, 2018](#)), but not their risk premiums as per the above. As a result, regressions of strategy returns on the UMVE result in low  $R^2$ s.

General equilibrium models have struggled with explaining FX moments. The current generation of models based on habit ([Verdelhan, 2010](#)), long-run risks ([Bansal and Shaliastovich, 2013](#); [Colacito, Croce, Gavazzoni, and Ready, 2018](#)), and rare disasters ([Farhi and Gabaix, 2016](#)) tend to aim at explaining the forward premium puzzle (and to some extent the carry trade). However, as our analysis shows, the most popular carry factors together explain only about 20% of the variation in our pricing factor. Further, the UMVE portfolio, which is perfectly negatively correlated with the SDF, has approximately zero skewness of  $-0.05$  and modest excess kurtosis of 3.07. This suggests a relatively modest effect of disaster risk pricing in our sample. Thus, there is much left to explain and our empirical model can help guide the specification and tests of future models on currency risk and return.

To that end, we first regress the UMVE returns on various candidate explanatory factors. We find that quarterly and 3-year consumption growth, a proxy for long-run consumption risk, both are significantly positively related to the UMVE returns, consistent with [Lustig and Verdelhan \(2007\)](#) and [Zviadadze \(2017\)](#). We next find that the factors in the Fama-French 5-factor model are only weakly related to the currency UMVE portfolio, with only the HML and CMA portfolios having a significant relation. However, here the  $R^2$ s are only about 1%, so economically there appears to be effectively no relation between priced risks in the equity and currency markets. These results extend findings in [Burnside \(2012\)](#), who shows a similar weak relation between the Carry trade returns and equity factors, to the UMVE portfolio. We also find that intermediary capital factors and shocks to equity and currency variance are unrelated to the UMVE returns. This may appear puzzling given that previous literature has highlighted these factors as potential explanations of the carry trade, but we show that this correlation comes from the large unpriced component in carry factor returns.

Finally, we show that the conditional price of risk, as given by the volatility of our currency market SDF, has a downward trend over the sample, largely caused by a trend decline in interest differentials across countries. The cyclical variation in the price of risk is *negatively* related to measures of the conditional variance of consumption and equity returns, and currency depreciation. In sum, our paper provides a rich set of facts that can help guide future developments of equilibrium models of currency market risk and return.

*Related literature.* A number of papers consider conditional mean-variance efficient portfolios (CMVE) using Carry alone as a signal ([Ackermann, Pohl, and Schmedders, 2017](#), [Baz, Breedon, Naik, and Peress, 2001](#), [Daniel, Hodrick, and Lu, 2017](#),

Maurer, To, and Tran, 2018, 2020). Further, with the exception of Maurer, To, and Tran (2018), these papers do not test if CMVE explains other versions of Carry or individual currency returns. We reject this version of our model with respect to our full model, which includes Value and Momentum, using the Barillas and Shanken (2017) test. We also reject it using the conditional pricing test, even when using Carry-based strategies only as test assets. Della Corte, Sarno, and Tsiakas (2009) depart from the lone Carry signal by considering three monetary fundamentals variables. They perform an out-of-sample analysis of exchange rate predictability in a mean-variance framework where the allocation choice is between the risk-free asset and one of the three exchange rates they consider.

A large literature implements model-free SDF projections in the context of unconditional pricing of assets. Many authors use Hansen and Jagannathan (1991) bounds as a diagnostic of their models, but not as a standalone tool for SDF estimation. A variation on the unconditional version of the Hansen and Richard (1987) approach is that of entropy minimization advocated by Stutzer (1996) in the context of derivatives pricing. Ghosh, Julliard, and Taylor (2019) develop and apply this framework to cross-sectional asset pricing. Korsaye, Trojani, and Vedolin (2020), Orlowski, Sokolowski, and Sverdrup (2019), and Sandulescu, Trojani, and Vedolin (2020) are examples of international applications of this framework. We critically depart from this work by constructing real-time conditional SDF projections. In particular, that allows valuation of any trading strategies in the set of assets involved in the projection.



## 2 Linear factor models and exchange rates

Linear factor models are popular in empirical asset pricing, in general, and in currency pricing in particular. The pursuit of such a model has two ultimate objectives. The first objective, common across all of asset pricing, is to achieve correct conditional pricing. That is, find a stochastic discount factor (SDF),  $M_{t,t+1}$ , such that

$$E_t(M_{t,t+1}R_{t,t+1}^{ei}) = 0, \quad (1)$$

where  $R_{t,t+1}^{ei}$  is a one-period excess return on asset  $i$ .

The second objective is to find a parsimonious factor representation of the risk-return trade-off. To make an extreme argument, having as many factors as assets does not help with understanding risk pricing in the economy. Further, a parsimonious representation is helpful in developing economic explanations of risk pricing. That is because it offers a succinct set of quantitative benchmarks, which an equilibrium model should match.

### 2.1 Conditional MVE portfolio

The ultimate parsimonious factor model is a single-factor one. The projection approach of [Hansen and Richard \(1987\)](#) offers a way to construct such a factor via a CMVE portfolio (also, see [Cochrane and Hansen, 1992](#) and [Cochrane, 2004](#)). Stack excess returns  $R_{t,t+1}^{ei}$  on all assets into an  $N_t \times 1$  vector  $R_{t,t+1}^e$ . Then

$$R_{t,t+1}^C = (w_t^C)^\top R_{t,t+1}^e, \quad (2)$$

$$w_t^C = V_t^{-1} (R_{t,t+1}^e) E_t (R_{t,t+1}^e) \quad (3)$$

is a CMVE portfolio. This portfolio “prices” any combination of these assets. That is, a projected SDF

$$M_{t,t+1} = 1 - (R_{t,t+1}^C - E_t(R_{t,t+1}^C)) \quad (4)$$

satisfies Equation (1). That there is a single factor driving this SDF does not mean that it prices only one source of risk in the economy. The factor is capturing the correct combination of the various risks that is priced.

The issue with this approach is practical. Implementation requires estimation of the conditional covariance matrix and conditional mean of a potentially large number of assets. The literature has responded by searching for a small number of factors whose linear combination with constant loadings in the SDF price assets unconditionally. That is, researchers use a low-dimensional factor approach to approximate the *unconditional* MVE portfolio. See [Chernov, Lochstoer, and Lundebj \(2018\)](#) for a recent distillation of the approach.

The UMVE construction corresponds to finding the SDF with smallest variance that prices a given set of assets. As highlighted by [Sandulescu, Trojani, and Vedolin \(2020\)](#), there is a whole family of objective functions that delivers valid SDF estimates. Another prominent member of this family corresponds to minimization of an SDF’s entropy. In this case, the estimated SDF is equal to the inverse of the growth optimal portfolio (see, e.g., [Bansal and Lehmann, 1997](#), [Long, 1990](#), and references therein). Such an SDF has a number of attractive features. In particular, in the context of exchange rates, one can express the SDF in different currencies easily without requiring market completeness.

The drawback of this approach is that, in contrast to variance minimization, entropy-

minimizing portfolio weights are not available in closed form. Thus, implementing entropy minimization requires numerical optimization subject to the correct pricing of a given set of assets. The latter requirement makes enforcing correct conditional pricing as per Equation (1) difficult. That explains why most entropy-based estimations obtain SDFs that price correctly unconditionally,  $E(M_{t,t+1}R_{t,t+1}^i) = 0$ .

## 2.2 Exchange rates

Exchange rates differ from equities in two very important dimensions. First, the cross-section of “assets” is limited. Indeed, the largest data panel considered in the literature goes up to 39 currencies at any given point in time (e.g., [Hassan and Mano, 2019](#), [Lustig, Roussanov, and Verdelhan, 2011](#), [Menkhoff, Sarno, Schmeling, and Schrimpf, 2012a](#)), while most studies limit their attention to ten currencies representing developed economies.

Second, a voluminous literature on the forecasting of exchange rates suggests that there are few variables that outperform the random walk hypothesis (RWH) out of sample. That observation, perhaps paradoxically, simplifies the modeling effort required to construct conditional expectations. We exploit these differences from the equity literature and construct a projection of the SDF on currency excess returns directly via Equation (4). Before we show how we do it, we introduce notation and basic concepts pertaining to exchange rates.

Let the USD be the base and measurement currency. Let  $S_t^i$  and  $F_t^i$  denote the spot exchange rate and the one-month forward exchange rate of country  $i$ , respectively. All exchange rates are expressed in USD per unit of foreign currency.  $M_{t,t+1}$  is the nominal USD-denominated SDF. Superscript  $i$  refers to a foreign country  $i$ .

The payoff of a forward contract (when buying one unit of the foreign currency) is  $S_{t+1}^i - F_t^i$ . One common way to scale this payoff to define excess return is to divide by  $F_t^i$ :

$$R_{t,t+1}^{ei} = (S_{t+1}^i - F_t^i)/F_t^i. \quad (5)$$

This definition implies that the amount of foreign currency bought is one “forward” dollar. Thus, this is an excess return to a trading strategy regardless of whether Covered Interest Parity (CIP) holds.

Next, we proceed with estimation of  $E_t(R_{t,t+1}^e)$  and  $V_t(R_{t,t+1}^e)$ . We use daily data within each month to construct our estimate of  $V_t(R_{t,t+1}^e)$  by combining the shrinkage method of [Ledoit and Wolf \(2020\)](#) along with an exponential moving average. See [Appendix A.1](#) for details.

Our starting point for  $E_t(R_{t,t+1}^e)$  is the Random Walk Hypothesis (RWH) for spot rates. The reason for this is simple: the hypothesis leads to an exceptionally robust forecaster of exchange rates (e.g., [Meese and Rogoff, 1983](#)). This is also a natural straw man given the research in finance, starting with [Baz, Breedon, Naik, and Peress \(2001\)](#), that uses RWH to develop trading strategies.

The RWH implies that expected excess currency returns are given by:

$$E_t(R_{t,t+1}^{ei}) = \gamma \cdot (S_t^i/F_t^i - 1)$$

with  $\gamma = 1$ . This is a particular violation of Uncovered Interest Parity (UIP), which posits  $\gamma = 0$ . Next, we add mean-reversion and trend signals for exchange rate forecasting. See [Bilson \(1984\)](#) and [Sweeney \(1986\)](#), respectively, for early contributions

in the context of trading strategies. Thus, the use of the RWH, as well as mean-reversion and trend signals, were all concepts that were in the public domain at the start of our sample in 1985.

Our mean-reversion signal is motivated by the literature on the role of real exchange rates (RER) in forecasting and capturing risk premiums. The RER is defined as

$$Q_t^i = S_t^i \cdot P_t^i / P_t, \quad (6)$$

where  $P_t$  and  $P_t^i$  are the U.S. and foreign price levels, respectively. The weak form of Purchasing Power Parity (PPP) implies mean-reversion in the RER. Thus, when the RER is far from its long-run mean it should forecast the currency depreciation.

Combining the RER-based signal with the RWH goes as far back as [Bilson \(1984\)](#). More recently, [Chernov and Creal \(2020\)](#) and [Dahlquist and Penasse \(2021\)](#) show that PPP implies that RER forecasts nominal depreciation rate and that currency risk premium depends on the RER. [Eichenbaum, Johannsen, and Rebelo \(2020\)](#) demonstrate that RER outperforms random walk in forecasting of exchange rates at horizons beyond one year.

As [Jorda and Taylor \(2012\)](#) emphasize, the RER's long-run mean is not a clearly defined object empirically. We divide each RER by its 5-year smoothed lag (specifically the average RER from 4.5 to 5.5 years ago) as a way to remove the dependence on the long-run mean while still preserving the long-run nature of mean-reversion signals:

$$\tilde{Q}_t^i \equiv Q_t^i \cdot \left( \frac{1}{13} \sum_{j=-6}^6 Q_{t-60+j}^i \right)^{-1}.$$

Lastly, we cross-sectionally demean the signal at each time  $t$  to create a cross-sectional

ranking of “cheap” and “expensive” currencies. That is, our signal is

$$z_{it}^Q \equiv \tilde{Q}_t^i - \frac{1}{N} \sum_{i=1}^N \tilde{Q}_t^i. \quad (7)$$

This definition has the virtue of removing any time and currency fixed-effects. The specific form of the reference point is inspired by the cross-sectional model of [Asness, Moskowitz, and Pedersen \(2013\)](#), who, in their turn, select the specifics to be comparable with the equity and commodity literatures.

Our trend signal is motivated by the academic and practitioner literature on using moving averages in trading and forecasting exchange rates. In contrast to the macro literature, the finance literature suggests that using past performance could be fruitful in improving trading performance. See, e.g., [Kho \(1996\)](#), [Okunev and White \(2003\)](#), [Sweeney \(1986\)](#), and references therein. Motivated by this work, we add a simple trend signal, which is a one-year depreciation rate.

In summary, we forecast excess returns out of sample via:

$$E_t(R_{t,t+1}^{ei}) = \gamma_t \cdot (S_t^i/F_t^i - 1) + \delta_t \cdot z_{it}^Q + \phi_t \cdot S_t^i/S_{t-12}^i. \quad (8)$$

We set  $\gamma_t = 1$  to match the RWH baseline. The coefficients  $\delta_t$  and  $\phi_t$  are re-estimated every month using a panel of  $N_t$  exchange rates and an expanding sample. Estimating  $\gamma_t$  in a similar fashion does not change the results. This specification departs from the traditional forecasting exercises in the macro literature where all the coefficients are presumed constant.

## 2.3 Testing factor models

### Tests

Factor models of expected returns are commonly tested using standard regression-based methods. The conditional SDF projection in Equation (4) implies that  $\alpha = 0$  in the regression:

$$R_{t,t+1}^e = \alpha + \beta \cdot R_{t,t+1}^U + \varepsilon_{t,t+1}, \quad (9)$$

where,  $R_{t,t+1}^U$  is the UMVE portfolio,  $\varepsilon_{t,t+1}$  is an error term, and where the UMVE portfolio is derived from the CMVE portfolio in the SDF projection as follows:

$$R_{t,t+1}^U = (1 + \theta_t)^{-1} R_{t,t+1}^C, \quad (10)$$

where  $\theta_t$  is the maximal conditional SR squared:

$$\theta_t = E_t(R_{t,t+1}^e)^\top V_t^{-1} (R_{t,t+1}^e) E_t(R_{t,t+1}^e).$$

Alternatively, since  $R_{t,t+1}^C = (w_t^C)^\top R_{t,t+1}^e$ , we can write the UMVE portfolio as a function of the conditional mean and variance of the underlying set of currencies:

$$R_{t,t+1}^U = (1 + \theta_t)^{-1} E_t(R_{t,t+1}^e)^\top V_t^{-1} (R_{t,t+1}^e) R_{t,t+1}^e. \quad (11)$$

See [Chernov, Lochstoer, and Lundebj \(2018\)](#) for derivation of these relationships.

The [Gibbons, Ross, and Shanken \(1989\)](#) test is a standard time-series joint test of  $\alpha = 0$  for all test assets in this setting. It is asking if it is possible to improve upon

SR of the candidate UMVE portfolio by forming a portfolio consisting of both the factor and the test assets. Obviously, this is always possible in sample and so the test accounts for sampling uncertainty.

The typical testing challenge in the equity literature is to find a set of test portfolios that are sufficiently informative about a given model (e.g., [Barillas and Shanken, 2017](#), [Daniel and Titman, 2012](#), [Lewellen, Nagel, and Shanken, 2010](#)). In the case of currencies, major cross-sectional and time-series factors that were proposed in the literature are natural test assets. Intuitively, factors that reflect dynamic trading strategies are particularly informative about conditional properties of the SDF.

Furthermore, we follow [Chernov, Lochstoer, and Lundebj \(2018\)](#) and use multihorizon returns (MHR) on the selected assets to generate additional test assets that are endogenous to the model of the SDF and pre-selected assets. Specifically, we test if

$$E(M_{t,t+h}R_{t,t+h}^i) = 1$$

for a range of horizons  $h$ .<sup>2</sup> The set of assets includes the same currency factors. In addition, we include MHR on the unconditional MVE. [Chernov, Lochstoer, and Lundebj \(2018\)](#) show that unconditional MHR pricing allows for testing most, if not all, aspects of conditional model misspecification.

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<sup>2</sup>The multihorizon SDF is  $M_{t,t+h} = \prod_{j=1}^h M_{t+j-1,t+j}$ . The multihorizon gross return is  $R_{t,t+h}^i = \prod_{j=1}^h R_{t+j-1,t+j}^i$ , where  $R_{t,t+1}^i = R_{t,t+1}^{ei} + R_{t,t+1}^f$  with the latter denoting the one-month U.S. gross funding rate.



## Trading strategies

The portfolio excess return of a trading strategy (portfolio)  $j$  is:

$$R_{t,t+1}^{P,(j)} = \sum_{i=1}^{N_t} w_{it}^{(j)} R_{t,t+1}^{ei}, \quad (12)$$

where  $w_{it}^{(j)}$  is the weight in currency  $i$  in strategy  $j$  and  $N_t$  is the number of currencies at time  $t$ . The weight can be based on a signal,  $z_{it}^{(j)}$ , and chosen such that the portfolio has an exposure to the dollar or not. We use so-called rank weights for cross-sectional (CS) strategies and sign weights for time-series (TS) strategies. The CS strategies are dollar neutral; the TS strategies are dollar exposed. The rank and sign weights are used and discussed by [Moskowitz, Ooi, and Pedersen \(2012\)](#), [Asness, Moskowitz, and Pedersen \(2013\)](#), [Kojien, Moskowitz, Pedersen, and Vrugt \(2018\)](#), and others. An alternative to rank weights is “high-minus-low” portfolios with currencies being equal-, value-, or signal-weighted within the “high” and “low” portfolios (see, e.g., [Lustig, Roussanov, and Verdelhan, 2011](#)).

We consider nine leading strategies proposed in the literature. The Dollar strategy is an equal-weighted average of the individual currency ([Lustig, Roussanov, and Verdelhan, 2011](#), [Verdelhan, 2018](#)). The Dollar Carry uses the average forward discount,  $S_t^i/F_t^i$ , as a signal. Specifically, it goes long (short) all currencies when the average discount is positive (negative) ([Lustig, Roussanov, and Verdelhan, 2014](#)).

The CS-Carry uses the currency’s forward discount as a signal. Specifically, ranks are based on the signal and weights are based on the ranks according to

$$w_{it}^{(j)} = \kappa_t \left( \text{rank}(z_{it}^{(j)}) - N_t^{-1} \sum_{h=1}^{N_t} \text{rank}(z_{ht}^{(j)}) \right), \quad (13)$$

where the scaling factor  $\kappa_t$  makes the portfolio one dollar long and one dollar short (Kojien, Moskowitz, Pedersen, and Vrugt, 2018). The TS-Carry uses the sign of the currency's forward discount as a signal. It goes long (short) currencies with a positive (negative) discount (Burnside, Eichenbaum, Kleshchelski, and Rebelo, 2011, Daniel, Hodrick, and Lu, 2017).

We consider two CS-Momentum strategies, which use the currency's performance as a signal. The first, CS-Mom 1, uses the performance in the most recent month as a signal (Menkhoff, Sarno, Schmeling, and Schrimpf, 2012b, Burnside, Eichenbaum, Kleshchelski, and Rebelo, 2011) and the second, CS-Mom 12, uses the performance in the most recent year skipping the most recent month as a signal (Asness, Moskowitz, and Pedersen, 2013). Specifically, weights are based on the signal ranks according to Equation (13). We also consider two TS-Momentum strategies, which use the sign of the currency's recent performance as a signal. The first, TS-Mom 1, uses the currency's last month performance (Burnside, Eichenbaum, and Rebelo, 2011) and the second, TS-Mom 12, uses the currency's performance in the last twelve months as a signal (Moskowitz, Ooi, and Pedersen, 2012). They both go long (short) currencies with a positive (negative) performance. Similar to Moskowitz, Ooi, and Pedersen, 2012, the second strategy also scales the weights inversely with the conditional return volatilities (same estimates as those used in the construction of the UMVE portfolio, see below).

Lastly, CS-Value uses the Bilson (1984) signal in Equation (7), whereby a relatively low (high) real exchange rate today indicates that the currency is cheap (expensive). Specifically, weights are based on the signal ranks according to Equation (13) (Asness, Moskowitz, and Pedersen, 2013).

## Signals used for test portfolios vs the UMVE

Given that we are using similar signals to construct the UMVE as per Equations (8) and (11) and trading strategies as per Equations (12) and (13), it is natural to wonder if our approach would be successful mechanically. First, a direct comparison of these equations shows that the Carry/Value/Momentum signals are used differently. In particular, the UMVE uses the actual numerical values as opposed to signs or ranks used for strategies. Second, conditional expectations featuring in the UMVE are weighed using the covariance matrix of individual excess returns. The resulting portfolio weights are not only different, they relate to each other in a highly nonlinear fashion.

A deeper question is whether a combination of the test portfolios span the UMVE in practice since similar signals are used:

$$R_{t,t+1}^U = b_t^\top R_{t,t+1}^P.$$

Here  $R_{t,t+1}^P$  denotes the vector of the strategy excess returns that we consider. This view is similar to the approach of [Kozak, Nagel, and Santosh \(2020\)](#) in the context of equities. If the coefficient  $b_t$  is constant then,  $b^\top R_{t,t+1}^P$  is the UMVE,  $R_{t,t+1}^U$ . Alternatively,  $R_{t,t+1}^P$  might span the CMVE. Then one has to estimate  $b_t$  to recover the UMVE. This endeavor requires estimates of  $E_t(R_{t,t+1}^P)$  and  $V_t(R_{t,t+1}^P)$ , as emphasized by [Haddad, Kozak, and Santosh \(2020\)](#) for equities.

Our approach delivers estimates of  $E_t(R_{t,t+1}^e)$  and  $V_t(R_{t,t+1}^e)$ . Thus we can compute conditional expectation and variance of any strategy. That allows us to evaluate if the test portfolios span the UMVE either unconditionally or conditionally. We do so

in the course of our empirical analysis.

### The role of currency denomination

Our main analysis pertains to a USD perspective. The question is whether the empirical results are affected by this choice. In fact, there are two separate choices with respect to currency that one may make.

The first choice is about the measurement currency, or numeraire. An investor funding herself in Japanese Yen (JPY), and receiving a payoff in JPY, faces a different excess return than a USD-funded investor in the same currency  $i$ . The one “forward” dollar strategy described in Equation (5) is equivalent to  $1/(F_t^i/S_t^{\$})$  contracts purchased with JPY. Thus, JPY excess return on forward position in currency  $i$  is

$$R_{t,t+1}^{ei,\$} = R_{t,t+1}^{ei} \cdot S_t^{\$}/S_{t+1}^{\$}. \quad (14)$$

Unless financial markets are complete, the SDF projected on JPY-denominated excess returns is not equal to the SDF projected on USD-denominated excess returns, and then converted into JPY. As a result, the pricing of risks may be different depending on the measurement currency.

The second choice is about the base currency. All the CS strategies that we discuss are not exposed to the USD by construction. As regards the remaining strategies, one could contemplate a different base, e.g., exposure to the euro, even if the measurement currency continues to be USD. One of the big points in the literature is that the choice of USD as a base currency is intentional because of the USD specialness (e.g., [Lustig, Roussanov, and Verdelhan, 2014](#), [Maggiori, Neiman, and Schreger,](#)

2020).

Given the specialness of USD as the base currency, we address only the choice of the measurement currency. Daniel, Hodrick, and Lu (2017) show theoretically, in continuous time with normally distributed innovations, that the difference between expected excess returns from the two different measurement perspectives is related to the covariance between the depreciation rate between the two measurement currencies and the given strategy return. They argue that this covariance is small. We ask a related but different question whether our conclusions about the pricing of the individual currencies or strategies change when considering another measurement currency.

One approach to address this question is to construct a new SDF projection for each measurement currency and then test it as described above for the USD. We propose an alternative empirical strategy. We simply convert the USD UMVE into a new measurement currency, e.g., JPY,  $R_{t,t+1}^{U,\text{¥}} = R_{t,t+1}^U \cdot S_t^\text{¥}/S_{t+1}^\text{¥}$ , and test if it prices individual currencies,  $R_{t,t+1}^{ei,\text{¥}}$ , and strategies  $R_{t,t+1}^{P,\text{¥}}$ , expressed in JPY.

As noted earlier, if financial markets are complete, this strategy delivers the correct projection of the SDF onto JPY-denominated excess returns. Next, suppose markets are incomplete. That conversion is still useful to evaluate because it corresponds to an actual trading strategy where a JPY-funded investor pursues the USD UMVE. Further, if the JPY-denominated USD SDF prices JPY-denominated currencies and strategies correctly, it can serve as an estimate of the valid JPY SDF for the test assets we consider.

Ex ante, one should expect such a proxy for the JPY SDF to price assets correctly because excess returns are (approximately) not affected by the adjustment via a

depreciation rate. Indeed, continuing with Equation (14):

$$R_{t,t+1}^{ei,\mathbb{Y}} = (S_{t+1}^i/F_t^i - 1) \cdot S_t^{\mathbb{Y}}/S_{t+1}^{\mathbb{Y}} = e^{s_{t+1}^i - f_t^i - \Delta s_{t+1}^{\mathbb{Y}}} - e^{-\Delta s_{t+1}^{\mathbb{Y}}} \approx s_{t+1}^i - f_t^i \approx R_{t+1}^{ei}, \quad (15)$$

where lowercase letters denote logs. We use a first-order approximation around zero for both  $s_{t+1}^i - f_t^i$  and  $\Delta s_{t+1}^{\mathbb{Y}}$ . Thus, the risk premium measured by the covariance between excess returns on the UMVE and on a candidate strategy should not be affected by the measurement currency either.

This result is reminiscent of excess returns on stocks, which are nominal assets, to be viewed as real. In that case inflation cancels out according to the log approximation. The difference between inflation and depreciation rates is the second-order effect of the approximation (convexity) because the latter are more volatile. Thus, we check empirically if our conjecture holds. We use JPY here as a specific example for exposition purposes. In the actual analysis we consider all currencies for measurement.

## 3 Evidence

### 3.1 Data

We construct a dataset of daily spot and one-month forward exchange rates expressed as USD per unit of the foreign currency for the G10 currencies (AUD, CAD, EUR spliced with DEM, JPY, NZD, NOK, SEK, CHF, GBP, USD) from January 1, 1976 to May 31, 2020. [Daniel, Hodrick, and Lu \(2017\)](#) offer compelling arguments for exclusion of emerging currencies and the European currencies other than the EUR. In particular, emerging currencies reflect credit risk of the respective sovereigns and,

therefore, the economic composition of the associated currency risk premiums is different (see, e.g., [Chernov, Creal, and Hördahl, 2020](#), [Na, Schmitt-Grohe, Uribe, and Yue, 2018](#)).

We use data from several providers through Datastream. Our monthly dataset includes the last day of every month from the daily dataset. Forward exchange rates for the AUD and NZD are available from December 1984, which effectively determines that January 1985 is the common starting month for currency excess returns.

We also collect monthly consumer price indices (CPIs) from the OECD for the period January 1976 to May 2020. Only quarterly CPIs are available for the AUD and NZD, so we forward fill the quarterly values to get monthly observations. As CPI is published with a lag and we want to ensure that all the variables are observable at time  $t$ , we construct the RER in Equation (6) using the price level lagged by three months.

The returns on the trading strategies are constructed using data going as far back as possible. All strategies use all nine currencies versus the USD from January 1985. Note that since both the value and momentum signals rely on spot rates, for which we have data going back to 1976, we have ten years of data to estimate the first conditional means (per Equation (8)) and covariance matrix for January 1985 when the currency excess return sample starts. We then each month expand the sample by one month to update these estimates in an out-of-sample fashion.

## 3.2 Preliminary evidence

Before we proceed with testing the model, we check if the main objects that we use for constructing the UMVE,  $E_t(R_{t,t+1}^{ei})$  and  $V_t(R_{t,t+1}^{ei})$ , are plausible. Specifically, we check if they predict  $R_{t,t+1}^{ei}$  and  $(R_{t,t+1}^{ei} - E_t(R_{t,t+1}^{ei}))^2$ , respectively.

Table 1A demonstrates results of regressing  $R_{t,t+1}^{ei}$  on  $E_t(R_{t,t+1}^{ei})$  and  $(R_{t,t+1}^{ei} - E_t(R_{t,t+1}^{ei}))^2$  on  $V_t(R_{t,t+1}^{ei})$  for individual currencies in a panel setting. Slopes in both regressions are significantly different from 0 and insignificantly different from 1. Even point-wise these coefficients are pretty close to 1: 0.84 and 0.93, respectively.

Panel B performs the same analysis for strategies. The conditional expected return and variance of each strategy are computed using the expected return vector and covariance matrix for the underlying individual currencies, along with the conditional portfolio weights of each strategy. The conclusion is the same as for individual returns. Because the strategies involve multiple returns, the results suggest that covariances of returns are estimated well.

Table 1C documents time variation in  $E_t(R_{t,t+1}^{ei})$  and  $V_t(R_{t,t+1}^{ei})$  for both individual currencies and strategies. In both cases, the variation in expected excess returns is substantial. For currencies, the volatility of conditional expectations is more than double than the average conditional expectation. For strategies, the two objects are about the same. As another perspective, conditional expectations are as variable as conditional variances.

Lastly, Panel D explores the role of the three signals that we used to construct conditional expectation. In the first row we report the unconditional SR for the full model, which equals 1.075. We emphasize that UMVE portfolio returns correspond



to a real-time implementable trading strategy. In the subsequent rows, we consider UMVEs formed on the basis of using the Carry signal alone, or Carry and Momentum, or Carry and Value. All of these combinations result in lower SR, albeit still high.

In order to assess the economic significance of using all the signals, we implement the [Barillas and Shanken \(2017\)](#) test and check if these three alternative UMVEs have an alpha with respect to the UMVE formed using all three signals. All of the alphas are significant. The corresponding adjusted  $R^2$ s vary between 50% and 80%. In unreported results, we reverse the direction of the regressions and find no significant alphas. Thus, the UMVE portfolio from the first row can explain UMVE returns from other versions of our model.

### 3.3 Testing the model's implications

#### Headline results

Table 2 presents our initial testing results. Panel A displays summary statistics for individual currencies (average excess returns and sample SR). SR for the UMVE reported in Table 1D exceeds the largest currency SR, NZD, by 2.5 times. The panel also shows alphas from individual regressions (9). The largest  $t$ -statistic is 1.67 for NZD. The largest adjusted  $R^2$  from regressing a return on the UMVE is 2.5% for AUD.

Panel B displays the same information for strategies. The UMVE SR exceeds the largest strategy SR, TS-Carry, by 1.6 times. None of the strategies have a significant alpha relative to the UMVE, with the highest individual  $t$ -statistic 1.75 for the Dollar

Carry strategy. On average, the alphas are about 70% smaller than the mean excess return on the strategies. Lastly, adjusted  $R^2$ s in these regressions are low, which implies that only a small component of the variation in these portfolios' returns is being priced. The Carry strategies and CS-Value exhibit the largest exposures to the UMVE with  $R^2$  around 15%, followed by Momentum around 9%, and the rest ranging between 0 and 6%.

The first row of Table 3, columns labeled “All strategies” display the GRS and MHR tests applied to the nine strategies. The MHR test uses strategy returns at the 1, 3, 6, 12, 24 and 48 month horizons. The  $p$ -values are 0.47 and 0.28, respectively. These results imply that the candidate UMVE prices single-horizon returns to the strategies both unconditionally and conditionally.

This failure to reject is meaningful. Consider a simpler model in which conditional expectations are computed on the basis of RWH/Carry alone and test it using the first five strategy portfolios, which do not rely on Momentum or Value signals. See the second row of Table 3, columns labeled “Dollar + All Carry”. GRS fails to reject with a  $p$ -value of 0.31, but MHR does reject with a  $p$ -value of 0.03. In other words, the UMVE based only on the conditional mean returns implied by the random walk hypothesis does not price long-run returns to the Dollar and Carry-based strategies.

### **Sources of portfolio timing**

To delineate the role of timing based on the conditional mean vs the conditional covariance, we also report in Table 3 test results for these sub-optimal models in rows three and four. The row labeled “ $V$ -weighted” corresponds to setting the conditional mean to a column of ones, while the row labeled “ $E$ -weighted” corresponds to setting

the conditional covariance matrix to the identity matrix. The SDF projection is not fully specified in these cases, so we cannot implement MHR tests. The column labeled “UMVE SR” documents a large drop in the SR as compared to the optimal portfolio weights. GRS strongly rejects both models.

### Alternative measurement currencies

Table 4 explores AUD and JPY as alternative measurement currencies. The results are qualitatively the same: strategy alphas are insignificant, adjusted  $R^2$  are low, and GRS  $p$ -values are high. The results for the remaining seven currencies are qualitatively similar and available upon request. That our results are not dependent on the choice of the measurement currency is not surprising in the light of Equation (15). Further, it indicates that the convexity effect is not quantitatively important.

To understand this result, we change the USD perspective that we entertain throughout the paper to the global perspective by changing the numeraire. Specifically, we choose a reference, denoted  $I_t$ , that is symmetric with respect to the exchange rates considered in this paper, the so-called world currency. Taking the cue from the USD index, known as USDX or DXY, we consider the geometric average of exchange rates,  $I_t = \prod_{i=1}^9 (S_t^i)^{\omega_i}$ . We take the weights  $\omega_i = 1/9$ . This definition of the index is consistent with the currency basket factor studied by [Korsaye, Trojani, and Vedolin \(2020\)](#).

Now we can express all the returns in terms of the numeraire  $I_t$  rather than USD. In particular, the UMVE returns are  $R_{t,t+1}^{U,I} = R_{t,t+1}^U \cdot I_t/I_{t+1}$  and currency returns are  $R_{t,t+1}^{ei,I} = R_{t,t+1}^{ei} \cdot I_t/I_{t+1}$ . Thus, one can think of returns in each local currency as consisting of two components. One is the returns expressed in terms of the world

currency numeraire. The second one that is due to the differences between the world numeraire and the local one, which is the currency of a specific country, e.g., USD or JPY. This second component is equal to  $I_{t+1}/I_t \cdot S_t^i/S_{t+1}^i$ . This perspective connects to global/local factor structure analyzed in [Lustig, Roussanov, and Verdelhan \(2011\)](#), [Korsaye, Trojani, and Vedolin \(2020\)](#), and many others.

To evaluate the role of each component, we can follow the same strategy as earlier and test whether the UMVE  $R_{t,t+1}^{U,I}$  can price strategy returns expressed in terms of  $I_t$ . Table 5 presents the evidence. It is essentially the same as the one obtained using local numeraires, such as USD in Table 2, or AUD and JPY in Table 4. This evidence reinforces our assertion that our approximation to the SDF in local currencies is not affected by the potential market incompleteness.

### Unpriced components of returns

The low  $R^2$  of strategy returns in Table 2 suggests a large unpriced component of these returns. [Daniel, Mota, Rottke, and Santos \(2020\)](#), in the context of equity strategies, make a strong case for hedging out the unpriced component to enhance strategy performance. It is easy to construct a conditional hedge in our setting because we have an explicit UMVE portfolio.

In particular, we use the conditional covariance matrix of the underlying currencies to construct the conditional beta of each strategy on the UMVE:

$$\beta_t = \frac{w_t^\top V_t(R_{t,t+1}^e)w_t^U}{(w_t^U)^\top V_t(R_{t,t+1}^e)w_t^U},$$

where  $w_t$  is a vector of strategy weights  $w_{it}$  as used in Equation (12). A portfolio

with systematic exposure, a.k.a. the hedged portfolio, is simply  $\beta_t R_{t,t+1}^U$ . We refer to the residual as the hedging portfolio return:

$$R_{t,t+1}^{HP} = R_{t,t+1}^P - \beta_t R_{t,t+1}^U.$$

We note that this hedging strategy is implementable in real time.

Figure 1 compares the SR of the original strategies to those of the hedging and hedged components. The SR for the hedging components are close to zero as should be the case for the unpriced piece if our model is well-specified. The SR for the hedged components are much larger than those of the original strategies as anticipated. For instance, for CS Carry, the SR goes from about 0.5 to slightly higher than 1. The one exception is the Dollar strategy where hedging does not seem to make much of a difference.

Table 3 reports GRS tests applied to the hedged portfolios in columns labeled “GRS hedged”. Specifically, we jointly test whether the average hedged returns to the strategies are zero, as implied by the model. The use of the conditional betas means that this is an unconditional test of the models’ conditional implications. The approach is similar in spirit to that undertaken in [Lewellen and Nagel \(2006\)](#) but where the pricing factor is the out-of-sample UMVE portfolio. Overall, this conditional test yields results similar to the unconditional GRS test: only our full model is not rejected in any of these tests.

A natural question related to the low  $R^2$  of returns is whether there is some common source of variation in the unpriced components of strategy returns. We implement principal component (PC) analysis of the hedging components of currency returns and those of strategy returns. We find, in both cases, that there is no dominant

component as the first PC explains 30% and 20%, respectively.

### 3.4 Time series implications

We analyze the factor structure implied by our model by performing PC analysis on the covariance matrix of currency returns. In this section, we consider the unconditional covariance matrix of individual currencies. Figure 2A displays the contributions of each of the nine PCs to the overall variation. The first PC explains 58%, the second 18%, and the third 9% of the variation.

Panel B shows loadings on individual returns on the first three PCs. We see that the first principal component loads similarly on each currency suggesting that it is related to the Dollar strategy. Indeed, the correlation between this first PC and the Dollar factor is 0.998. The second PC goes long the high interest currencies and goes short the low interest currency and is thus related to Carry – their correlation is 0.718. The third PC is harder to associate with an extant strategy, though a possible interpretation is geographical with largely positive loadings for European countries and largely negative for the other. These results are consistent with [Verdelhan \(2018\)](#) who argues that Dollar and Carry capture exchange rate movements.

Panel C explores how much of the UMVE portfolio’s variation can be explained by the PCs. We start by repeating the information of Panel A. Rather than reporting the individual contribution of each PC, we now report their cumulative contribution. Next, starting with the first PC and gradually adding them one after another, we compute how much of the variance of UMVE returns can be explained by a given number of PCs.

The answer is not much. It is close to zero if we use the first PC alone. Recall that this PC explains almost 60% of the variation in individual currency returns. If we use all nine PCs, we progress to about 10% of the variation. This is consistent with low  $R^2$  reported in Table 2B. The flipside of this result is that UMVE has a significant alpha with respect to the PCs as evidenced in Figure 2D.

We conclude that time-series variation in currency returns can be summarized by three factors, two of which are close to the Dollar and CS-Carry strategies. However, these factors are weakly related to the UMVE, which prices the cross-section of both currencies and strategies. That suggests that factor timing is an important ingredient of the pricing success. We proceed with fleshing that out in the next section.

### 3.5 Factor timing

The leading currency strategies, which we study in this paper involve a substantive amount of currency timing. Thus, a natural starting point is to investigate how well these trading strategies can explain UMVE portfolio returns. Figure 3 addresses this issue via PC analysis of strategies. This analysis parallels that in Figure 2.

Panel A shows individual contributions of PCs to the overall variation in strategy returns. Here three PCs explain 65% of the variation. Panel B displays the loadings of these three PCs on strategy, but no clear interpretation of what these “strategies on strategies” might entail emerges. Panel C shows how much of the variance of UMVE returns can be explain by strategy PCs. The first three PCs explain only about 20% of the variation in UMVE returns. Thus, again the main drivers of the time series variation asset returns are not the main drivers of priced risk, as summarized by the

UMVE. The first five PCs can explain about 50% of the variation and this number stays the same after adding the remaining PCs.

The improvement from 10% to 50% is a testament to the importance of currency timing implicit in the strategies. Yet, there is a lot left on the table. UMVE alphas with respect to these PCs continue to be significant as documented in Panel D.

One might wonder if the strategy returns span the CMVE and thus conditional regressions might reveal a closer connection between the strategy returns and the UMVE. If the nine strategies considered in this paper were conditionally non-collinear, then, because strategy weights are known, one could re-express the UMVE as a portfolio of strategies. The weights in such portfolio would be changing every period, so one would still have to establish optimal portfolio weights in terms of currency returns, or alternatively, directly model conditional expected returns and covariance matrix of the strategies themselves.

In practice, some of the strategy returns are closely related to each other. For instance, the Dollar Carry is conditionally perfectly correlated with the Dollar. Likewise, the different Carry and Momentum strategies are related to each other. Thus, we cannot construct a portfolio of strategies that tracks the UMVE perfectly.

Instead, we use four strategies and strive to get as close to the UMVE as possible. The four strategies are: the Dollar, the sum of the CS- and TS-Carry, the sum of the four Momentum strategies, and CS-Value. Next, we apply Equations (2), (3) and (10) to strategy returns as opposed to the CMVE returns. We can implement this equation because our model produces real-time conditional expectations and variance for each currency and the portfolio weights for each strategy are known.



Regressing the UMVE-tracking portfolio on our UMVE portfolio gives an  $R^2$  of 84%. Its SR is 0.93 vs 1.07 for the UMVE portfolio. The [Barillas and Shanken \(2017\)](#) test rejects the null that the UMVE-tracking portfolio can explain the UMVE with a  $p$ -value of 0.2%. Thus, we cannot replicate the optimal portfolio with extant trading strategies.

Despite the failure to track the UMVE, it is still informative to study the properties of the resulting strategies-based proxy. Figure 4 displays the 12-month moving averages of the weights for the four strategies in this tracking portfolio. As a reference point, [Asness, Moskowitz, and Pedersen \(2013\)](#) famously suggest to consider 50% in Momentum and 50% in Value. In that spirit, the sample averages of the strategy weights are positive. In contrast to the [Asness, Moskowitz, and Pedersen \(2013\)](#) recommendation, the optimal strategy weights have strong time variation. Indeed, the ratio of the sample standard deviation of the weights divided by the sample mean is 1.21.

Figure 5 evaluates the economic significance of the documented time-varying weights by comparing the UMVE-tracking portfolio to alternative portfolios of the same four strategies. In addition to the 50% in Momentum and 50% in Value portfolio, we consider equal-weighted (25% in each of the four strategies), and a variance timing strategy that uses the inverse of the conditional variance of each strategy (obtained from our conditional covariance matrix estimate) as the portfolio weights. We display time series of the difference between conditional SR of the UMVE-tracking portfolio and one of these three alternatives. The differences in SR are large, ranging between 0.32 for equal-weighted and 0.42 for Momentum-Value, on average (annualized).

According to the [Barillas and Shanken \(2017\)](#) test reported in Panel (D), the fixed-

weight strategies cannot explain the UMVE-tracking portfolio, i.e., the latter has significant “alpha” with respect to the former. The latter *can* explain the former despite being a suboptimal proxy of the UMVE. In sum, there is substantial time-variation in the conditional mean and variance to the strategy returns that leads to significant gains for a mean-variance investor from timing these portfolios.

### 3.6 Interpreting the evidence

We follow [Lustig, Roussanov, and Verdelhan \(2011\)](#) and use an affine no-arbitrage model as an organizing framework for interpreting our evidence. As a part of such a model, we switch to logs from levels. For clarity of exposition, we ignore conditional means of variables. As a result, depreciation returns and currency excess returns coincide.

Recall that we have introduced the world currency index  $I_t$  in section 3.3. We assume that the log depreciation rate of country  $i$  relative to this numeraire is:

$$\Delta s_t^i = \beta_{it-1} u_t + \gamma_i v_{it}.$$

Here innovation  $u_t \sim N(0, 1)$  represents the UMVE portfolio expressed in terms of  $I_t$  and innovation  $v_{it}$  represents the component of depreciation rates that is orthogonal to  $u_t$ . Our PCA analysis in section 3.4 suggests that there are at least two common components in  $v_{it}$ : Dollar and Carry. Accounting for that fact we can write:

$$\gamma_i v_{it} = \delta_i f_t + \sigma_i \varepsilon_{it}, \tag{16}$$

where  $f_t$  represents these common sources of variation in  $v_{it}$  and  $\varepsilon_{it}$  are cross-

sectionally uncorrelated innovations. This representation also implies a depreciation rate between currencies  $i$  and  $j$ :

$$\Delta s_t^{ij} = \Delta s_t^i - \Delta s_t^j = (\beta_{it-1} - \beta_{jt-1})u_t + (\delta_i - \delta_j)f_t + \sigma_i \varepsilon_{it} - \sigma_j \varepsilon_{jt}.$$

The log SDF expressed in USD (currency 0) is:

$$m_{t,t+1} = -b \cdot u_{t+1} - \Delta s_{t+1}^0. \quad (17)$$

Then we can express the SDF in any other currency via

$$m_{t,t+1}^i = m_{t,t+1} + \Delta s_{t+1}^{0i} = -b \cdot u_{t+1} - \Delta s_{t+1}^i.$$

Thus, regardless of the perspective, SDF has a component that prices all currencies and strategies, and another component that reflects pricing in local currency.

To get a sense of the relative importance of the different pricing components consider the maximal SR of the USD SDF from Equation (17):

$$\begin{aligned} V(m_{t,t+1}) &= b^2 + V(\Delta s_{t+1}^0) + 2b \cdot V^{1/2}(\Delta s_{t+1}^0) \cdot \text{corr}(u_{t+1}, \Delta s_{t+1}^0) \quad \Rightarrow \\ |b - V^{1/2}(\Delta s_{t+1}^0)| &\leq V^{1/2}(m_{t,t+1}) \leq |b + V^{1/2}(\Delta s_{t+1}^0)|. \end{aligned}$$

Our evidence in Table 1D implies that the volatility of the SDF, as proxied by SR of the UMVE, is equal to 1.075. Volatility of the depreciation rate is about 0.1 per annum. Therefore,  $b \approx 1$ .

This back-of-the-envelope computation strongly suggests that the major share of pricing comes from  $u_{t+1}$ . This conclusion remains the same if we replace the estimate

of  $V^{1/2}(m_{t,t+1})$  using the strategy with the largest SR, which is TS-Carry. That explains why changing the measurement currency in Tables 4 and 5 did not affect our conclusions materially.

In order to understand the low  $R^2$  reported in Table 2, consider explicitly the strategy returns:

$$r_{t,t+1}^P = \sum_{i=1}^N w_{it} \cdot \Delta s_{t+1}^{0i}.$$

For example, if the strategy is the Dollar, portfolio weight are equal. Alternatively, if a strategy is USD-neutral, like all of our CS strategies,  $\sum_{i=1}^N w_{it} = 0$ .

Next, in the context of this model, we can define priced and unpriced components of returns that we characterize in our empirical work. Extending Equation (17), we obtain

$$m_{t,t+1} = -(b + \beta_{0t}) \cdot u_{t+1} - \delta_0 f_{t+1} - \sigma_0 \varepsilon_{0t+1}.$$

In the data we can measure this linear combination of the various innovations, each of which covaries with the SDF. The priced component of  $r_{t,t+1}^P$  is the one that covaries with this specific linear combination.

Now, considering the CS strategies, if  $N$  is sufficiently large,  $r_{t,t+1}^P$  does not depend on any of  $\varepsilon_{it+1}$ , but still depends on  $u_{t+1}$  and  $f_{t+1}$ . The component  $f_{t+1}$  is important for explaining variation in strategy returns as per our PCA analysis. Yet, it reflects the part of the variation in depreciation rates that is not exposed to  $u_{t+1}$ . Thus, as the discussion above indicates, this component cannot contribute much to risk premiums. In the case of the Dollar, strategy returns are also exposed to the USD

innovation  $\varepsilon_{0t+1}$ . Nevertheless, the reasoning remains the same:  $f_{t+1}$  and  $\varepsilon_{0t+1}$  jointly represent the fraction of variation in strategy returns not related to  $u_{t+1}$  and, thus, does not contribute much to risk premiums.

This logic explains how we obtain low values of  $R^2$ . We emphasize that this conclusion is specific to our evidence. For instance, if all  $\delta_i$ 's in Equation (16) were equal to zero, then CS strategy returns would be exposed to  $u_{t+1}$  only. In that case  $R^2$  from regressing returns of such strategies on the UMVE, which is represented by  $u_{t+1}$  here, would be equal to 1.

### 3.7 Properties of the projected SDF

The estimated UMVE corresponds to the linear projection of the SDF on the set of currency excess returns. Figure 6 plots time series of both the conditional annualized price of risk (the conditional annualized volatility of the SDF) and returns to the UMVE portfolio, which is perfectly negatively correlated with this SDF. The price of risk varies strongly over the sample, from a high of about 4 to a low of about 0.5.

We see that the price of risk experiences its maximum value in a dramatic move during the 1992 Exchange Rate Mechanism crisis. Outside of this event, the main take-away is a substantial decline through the sample, mainly caused by a trend decrease in interest differentials across countries. In particular, the same decline is shared by the price of risk of the model that only uses the RWH when estimating currency expected returns (red dashed line). Notably, recessions, showed as yellow bars on the plot, are associated with a *decline* in the price of risk.

To get a sense of whether these magnitudes are reasonable we also display ex post realized maximal SRs. Specifically, at each date  $t$  we use past 36 months of returns to compute sample SR for each strategy. Next, we select the largest SR. These SRs range between 0.15 and 2.8 and feature a similar declining pattern throughout the sample. These rough computations suggest plausibility of our estimated price of risk.

The bottom panel of Figure 6 shows the time series of returns to the UMVE portfolio (the dashed red line displays the RWH case). First, the returns are relatively smooth, with little evidence of a disaster event. For instance, the Global Financial Crisis is associated with small negative returns and one big *positive* spike. The sample skewness and excess kurtosis are  $-0.05$  and  $3.07$ , respectively. That suggests modest pricing of disaster risks in currency returns.<sup>3</sup> Second, we can see a slow decrease in the volatility in returns over the sample, consistent with the pattern in the price of risk.

As a next step, we make an attempt to relate these two objects to observable variables that have the potential to capture currency risk premiums. Following the literature, we focus on macro and financial variables. Among macro variables, we have a particular interest in consumption as it has a rich history and connection to equilibrium models. Among financial variables, we consider the [Fama and French \(2015\)](#) model as the de-facto SDF projection on on the space of equity returns. Finally, we consider measures of intermediary leverage. The full list of candidate covariates and motivation for considering them are offered in [Appendix A.2](#).

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<sup>3</sup>We emphasize that this conclusion is not a mechanical implication of how we project the SDF, that is, portfolio weights are constructed using only mean and variance of excess returns. Our portfolio is represented by a linear combination of excess returns. If these returns exhibit systematic exposure to crash risk, that will show up in the UMVE. Ex ante, or by construction, nothing prevents this from happening. If crash risk is important for risk pricing, assets that load on crash risk will have high SR and high weights in the UMVE.

Table 6 reports the results. Panel A displays univariate regressions for the significant covariates. We see that quarterly consumption growth and 3-year consumption growth, a proxy for the long-run consumption component, are significant. This result is consistent with [Lustig and Verdelhan \(2007\)](#) and [Zviadadze \(2017\)](#), who argue that these variables are important for explaining the CS-Carry premium. Our results generalize theirs because we find significant relation to the UMVE, and not just Carry returns. This generalization also explains why we find strong statistical significance. Focus on the UMVE removes the unpriced components and, thus, helps recover a more precise relationship with consumption growth as compared to a regression of strategy returns, like CS-Carry, on consumption. The  $R^2$ s of the regressions are only 3% and 7% indicating that other elements than consumption risk may be important for understanding the currency market conditional risk-return trade-off.

Panel A also shows that only two out of the five Fama-French factors are significant, value (HML) and investment (CMA). Even though they are significant, the relation to the UMVE returns is weak as betas are very small.  $R^2$ s are small as well, at 0.6% and 1.3%, respectively.

Panel B focuses on the properties of the CS-Carry, a strategy that has received a lot of attention both in the empirical and equilibrium literatures. The left section of the Panel, labeled ‘CS-Carry’, demonstrates that consistent with the literature returns to this strategy are significantly related to intermediary leverage ([He, Kelly, and Manela, 2017](#)), realized variance of equity ([Lustig, Roussanov, and Verdelhan, 2011](#)), and that of exchange rates ([Menkhoff, Sarno, Schmeling, and Schrimpf, 2012b](#)). However, as foreshadowed by Panel A, the priced component of CS-Carry returns are not related to these variables. Indeed, Panel B makes it explicit in the middle panel labeled ‘CS-Carry priced’. Results in Table 2B suggests reconciliation of that evidence.

Only 15% of CS-Carry is explained by the UMVE. Thus, it must be the unpriced component of the strategy that is related to these variables. Indeed, that is what we find (see the right section of Table 6B). This evidence can be reconciled with the extant literature in the context of omitted factors (Giglio and Xiu, 2020), or, more generally, misspecified factors (Jagannathan and Wang, 1998).

Lastly, Panel C relates prices of risk to various measures of aggregate uncertainty: conditional consumption variance, realized equity variance, and realized FX variance. The conditional variance of consumption growth is estimated using a GARCH(1,1) model. Due to the downward trend in the price of risk in the sample, we run these regressions in 3-year differences. The conditional price of risk, that is, conditional volatility of the SDF, is *negatively* related to consumption variance and FX return variance. Thus, periods of high uncertainty are associated with lower SR in the currency market. The sign is negative also for equity variance, though it is insignificant in this case. Figure 7 plots the price of risk vs. the conditional variance of consumption growth. The common trend and the negatively correlated cyclical relation are both clearly visible.

## 4 Conclusion

We construct a conditional linear projection of the SDF on the space of excess currency returns. This is feasible as most of the literature focuses on the G10 currencies and the corresponding nine exchange rates vs the U.S. dollar. A small cross-section makes estimation of the conditional covariance matrix relatively precise. We then use standard expanding panel regressions and a limited set of longstanding currency



characteristics to obtain conditional expected returns. These inputs are all that is needed to construct the projection.

The benefit of this approach is that it is a theoretically motivated empirical characterization of the risk-return trade-off. The use of the conditional covariance matrix means we also characterize the factor structure in realized returns, which allow us to discuss the priced versus unpriced sources of common variation in currency returns. Our analysis leads to a number of new insights.

First, the UMVE portfolio has a sample SR in excess of one and is not explained by any existing factors. At the same time, the UMVE portfolio accounts for known currency factors risk premiums and is not rejected as the pricing factor in standard model tests. Thus, our proposed estimation methodology is validated in the data and uncovers heretofore undiscovered sources of risk.

Second, we show that most of the common variation in currency returns is due to unpriced risks – that is, factors that do not command a risk premium. We show that existing currency strategies indeed have large uncompensated components that can be hedged away to obtain much higher Sharpe ratios.

Third, we document large gains to timing many of the existing factors, owing to the fact that the conditional dynamics of these factors are strongly time-varying. In fact, the standard deviation of conditional risk premiums is as large as the level of unconditional risk premiums.

Fourth, the priced currency risks are effectively unrelated to priced risks in the equity market, as well as shocks to equity and currency market return variance and intermediary capital. Consumption growth, however, is related to priced risks,

though the relation is not economically very strong. That said, our results support a role for consumption risk in understanding currency risk and return.

Fifth, the conditional price of risk is strongly downward trending through the sample, with a cyclical component that is *negatively* related with measures of the conditional variance of macroeconomic aggregates and market returns.

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# Appendix A Appendix

## Appendix A.1 Estimating covariance matrix

First, for each month  $t$ , we compute the realized covariance matrix based on daily depreciation rates. That is, define  $X_{j,t}$  as the vector of percentage changes in spot currency rates over day  $j$  in month  $t$ . The realized covariance matrix is then

$$\hat{\Sigma}_t = \sum_{j=1}^{D_t} X_{j,t} X_{j,t}^\top$$

where  $D_t$  is the number of trading days in the month. Given a potentially large cross-section and short time-series, we apply the shrinkage method of [Ledoit and Wolf \(2020\)](#) to  $\hat{\Sigma}_t$  to improve the mean-squared-error of this sample covariance estimate. Denote the resulting shrunk matrix as  $\tilde{\Sigma}_t$ .

Second, we apply a simple exponentially weighted average to  $\tilde{\Sigma}_t$  to arrive at the estimate of the conditional covariance matrix for month  $t + 1$  currency percentage price changes:

$$V_t = (1 - \lambda)\tilde{\Sigma}_t + \lambda V_{t-1},$$

where we set  $\lambda = 0.94$  following, e.g., the RiskMetrics model. Finally, to get to the conditional covariance matrix of excess currency returns, we pre- and post-multiply this matrix by a diagonal matrix with  $(i, i)^{th}$  element set to  $S_t^i/F_t^i$ .

## Appendix A.2 Additional data sources

This appendix lists and motivates the macroeconomic and financial variables used as covariates in Section 3.6 to characterize the properties of the projected SDF. We consider several groups of covariates.

The first group of covariates are based on a per capita series of US real consumption expenditure on nondurables and services that we construct from the NIPA Table 7.1. at the Bureau of Economic Analysis. The data and more detailed information about the series are available through the webpage:

[https://apps.bea.gov/iTable/index\\_nipa.cfm](https://apps.bea.gov/iTable/index_nipa.cfm).

We construct quarterly log consumption growth, three-year log consumption growth, and consumption growth volatility on a quarterly basis from 1985:Q1 to 2020:Q1. These consumption growth measures are appealing as they naturally relate to equilibrium models of asset prices. Several studies have attempted to explain currency returns or trading strategies to short-term consumption growth (Cumby, 1988; Sarkissian, 2003; Lustig and Verdelhan, 2007), long-term consumption growth (Colacito and Croce, 2011), and consumption growth volatility (Lustig, Roussanov, and Verdelhan, 2014).

The second group of covariates are the equity factors in the Fama and French (2015) five-factor model. The data are available through the webpage:

[http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data\\_Library/](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/). More specifically, the equity factors are: the return on a world market portfolio minus the U.S. one-month T-bill rate (market, MKT); the return on a small stock portfolio minus the return on a big stock portfolio (small minus big, SMB); the return on a value portfolio minus the return on a growth portfolio (high minus low, HML); the return on a robust-operating-profitability portfolio minus the return on a weak-operating-profitability portfolio (robust minus weak, RMW); and the return on a conservative investment portfolio minus the return on an aggressive investment portfolio (conservative minus aggressive, CMA). We use monthly observations from January 1985 to May 2020. Examples of studies relating carry trades to equity factors include Lettau, Maggiori, and Weber (2014) and Daniel, Hodrick, and Lu (2017).

The third group of covariates are based on the literature of financial intermediaries and their leverage (Adrian, Etula, and Groen, 2011; Adrian, Etula, and Muir, 2014; He, Kelly, and Manela, 2017). More specifically, we consider the monthly intermediary capital measures of He, Kelly, and Manela (2017) from January 1985 to May 2020. The data are available through the webpage:

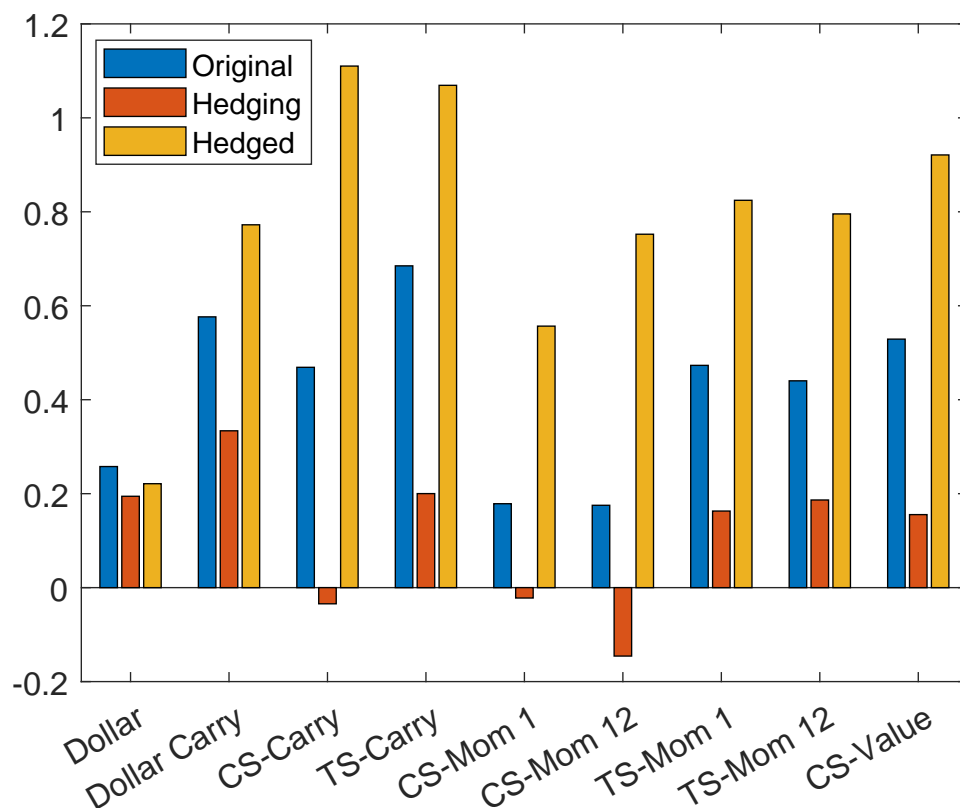
<http://apps.olin.wustl.edu/faculty/manela/data.html>.

Adrian, Etula, and Groen, 2011 decompose currency risk premiums into components associated with macroeconomic and balance sheet variables. He, Kelly, and Manela (2017) explain currency trading strategies based on cross-sectional carry and momentum.

The fourth group of covariates are based on measures of global equity and foreign exchange rate volatility. Lustig, Roussanov, and Verdelhan (2011) and Menkhoff, Sarno, Schmeling, and Schrimpf (2012a) find that cross-sectional carry strategies

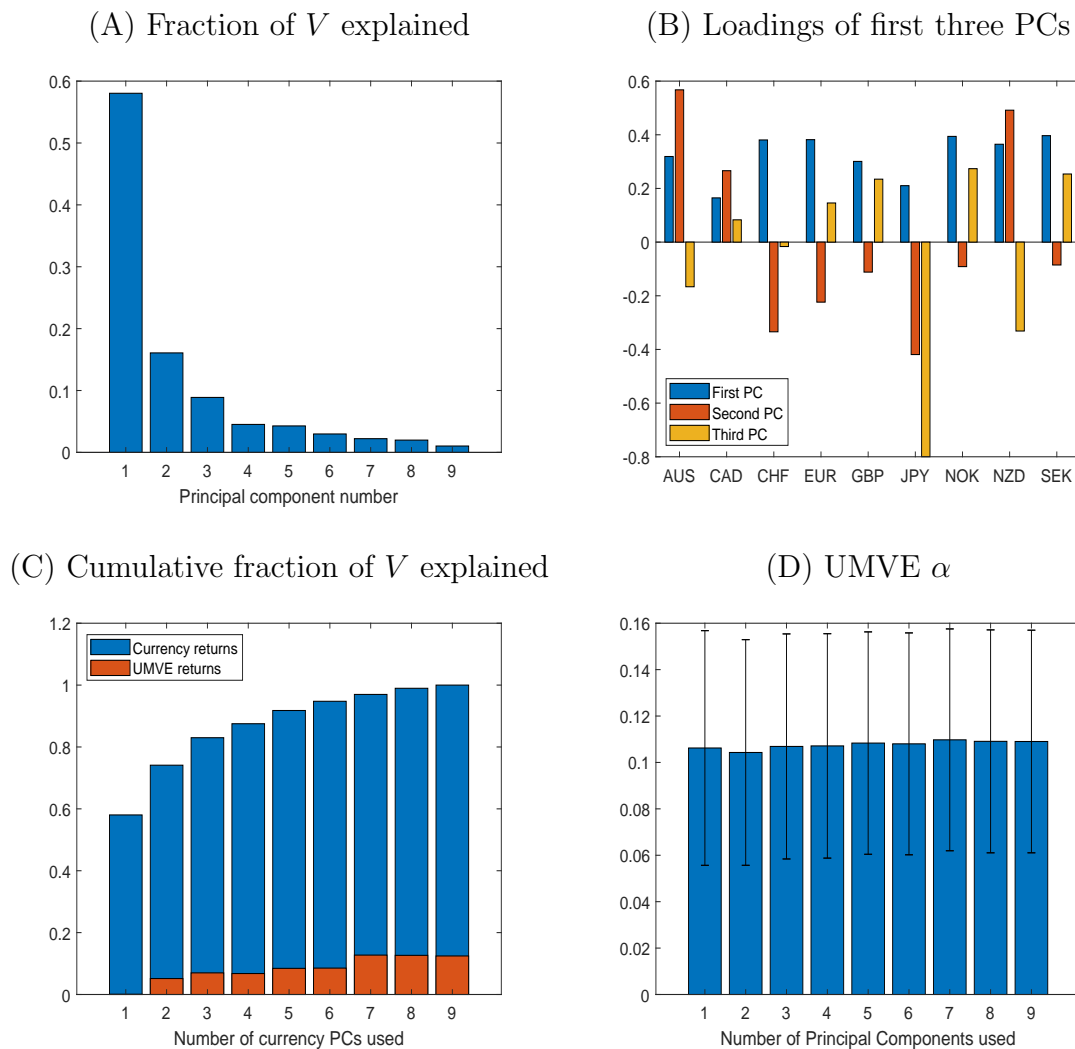
relate to global equity and foreign exchange rate volatility, respectively. We construct monthly realized variances from daily changes in equity prices and spot exchange rates similar to these authors. We consider the period January 1985 to May 2020.

**Figure 1**  
**Sharpe ratio of original, hedging, and hedged strategies**



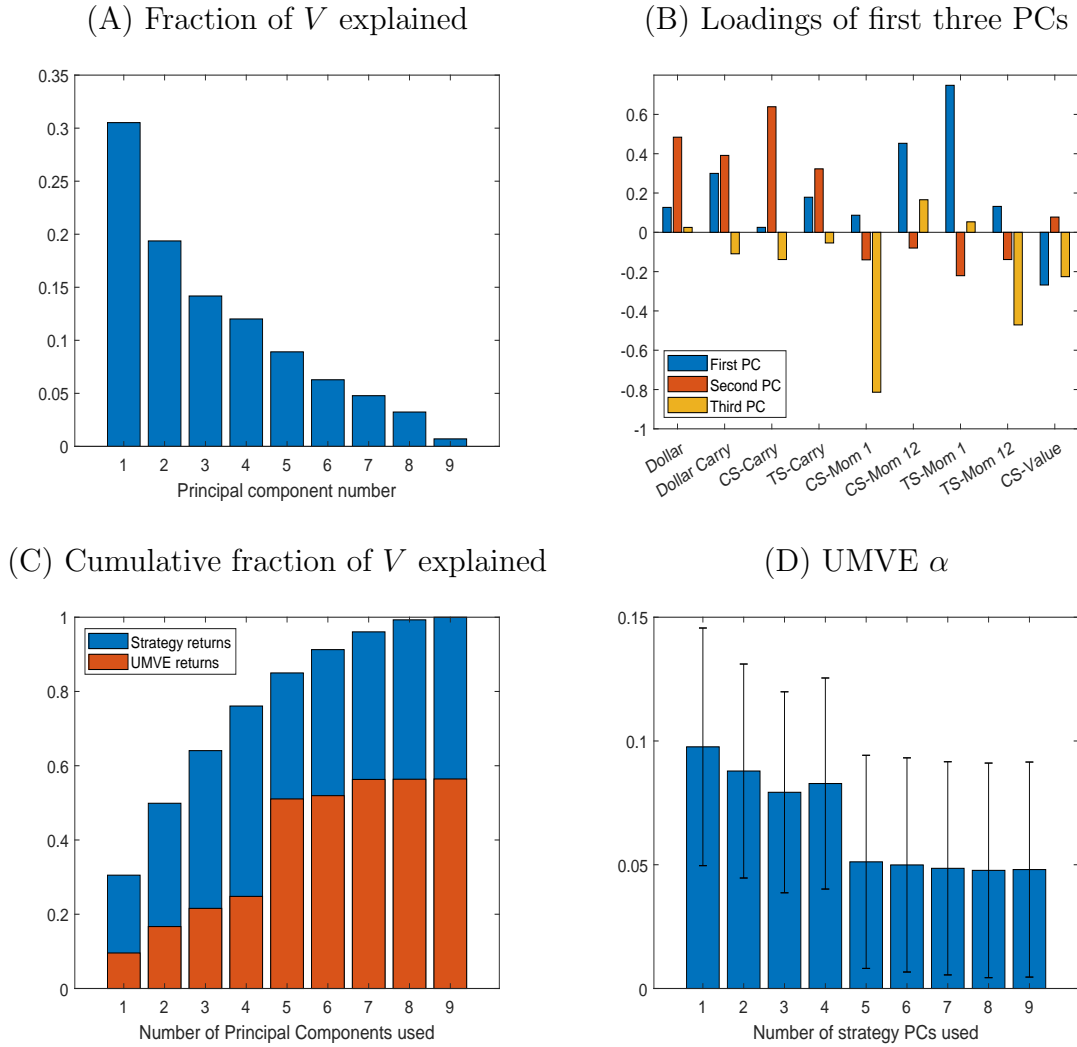
The blue bars show the sample annualized Sharpe ratios of each strategy. The red bars shows the same for each strategy's hedging portfolio, defined as the component of the strategy returns that is unpriced according to the model. The yellow bars show the Sharpe ratio of the factor returns when unpriced risks are hedged out. All portfolios are tradeable in real time. The sample is monthly from January 1985 to May 2020.

**Figure 2**  
**Principal component analysis of currencies**



Panel A shows the fraction of variance across the nine individual currencies that each principal component (PC) explains. The PCs are obtained from the unconditional covariance matrix of individual currency returns. Panel B shows the loadings of the three first PCs on each currency. Panel C shows in blue the cumulative amount of currency variance explained as one goes from using one to all nine PCs. In red is the  $R^2$  of a regression of the UMVE returns on an increasing number of PCs. Panel D shows the “alpha” of a regression of the UMVE returns on an increasing number of PCs. The error bars correspond to the 95% confidence interval computed using heteroskedasticity-adjusted standard errors. The sample is monthly from January 1985 to May 2020.

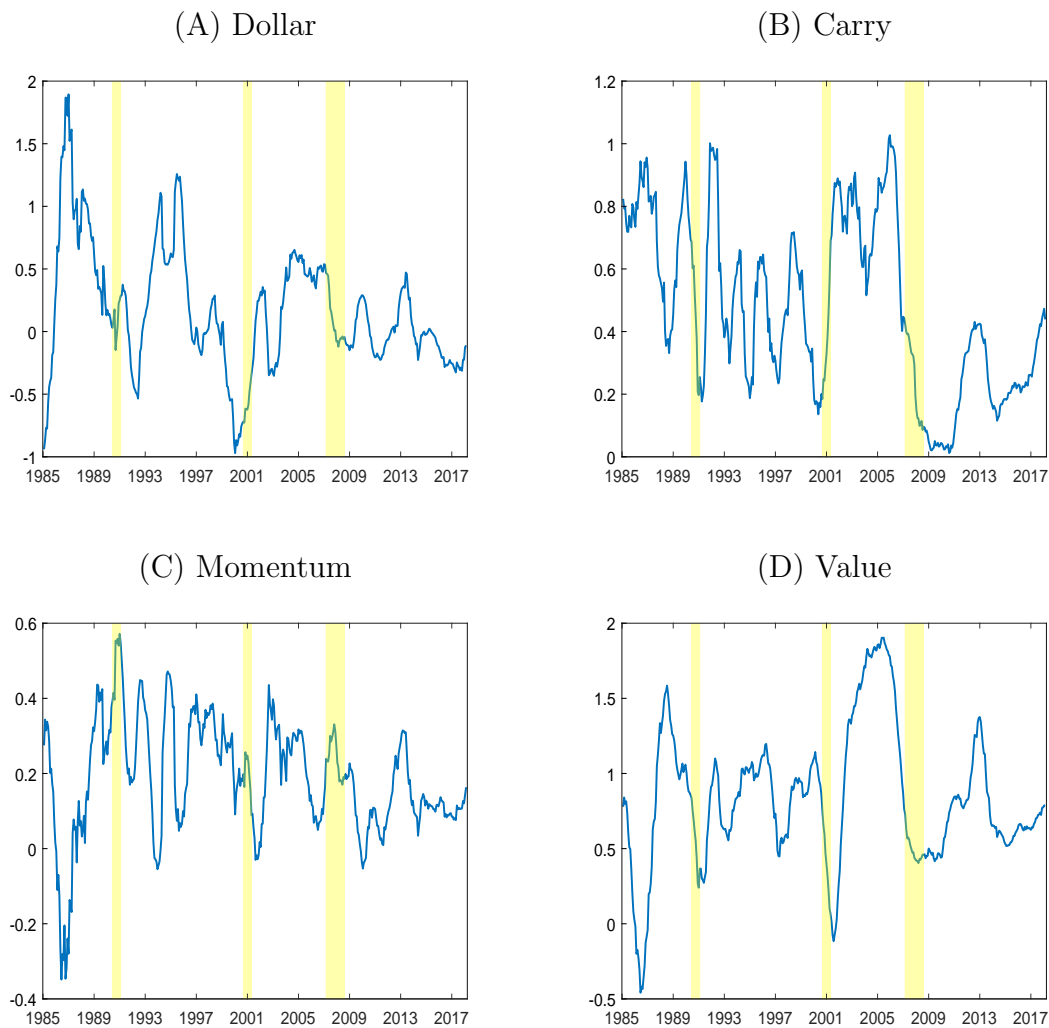
**Figure 3**  
Principal component analysis of strategies



Panel A shows the fraction of variance across the nine currency trading strategies that each principal component (PC) explains. The PCs are obtained from the unconditional covariance matrix of these currency strategies. Panel B shows the loadings of the three first PCs on each strategy. Panel C shows in blue the cumulative amount of strategy variance explained as one goes from using one to all nine PCs. In red is the  $R^2$  of a regression of the UMVE returns on an increasing number of these PCs. Panel D shows the “alpha” of a regression of the UMVE returns on an increasing number of PCs. The error bars correspond to the 95% confidence interval computed using heteroskedasticity-adjusted standard errors. The sample is monthly from January 1985 to May 2020.

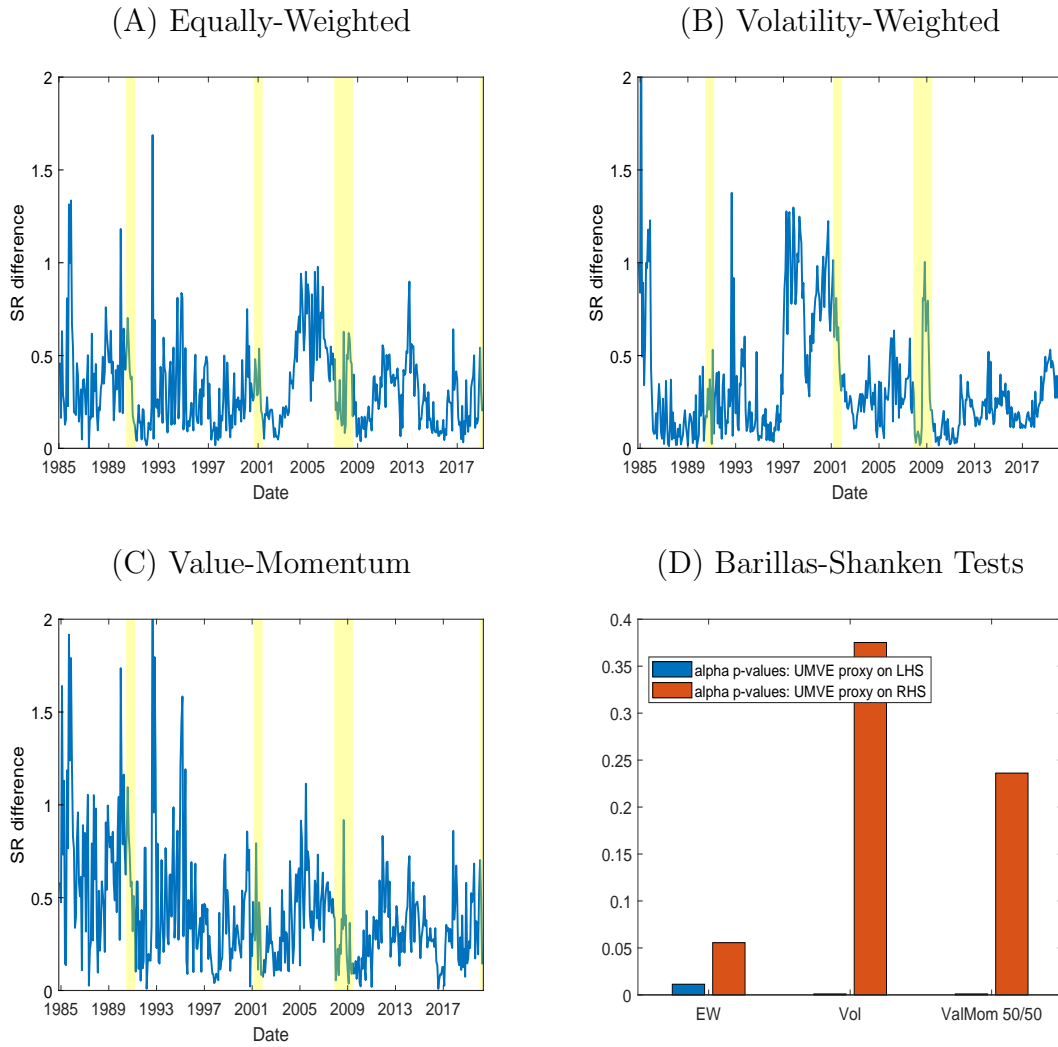


**Figure 4**  
**The UMVE tracking with strategies**



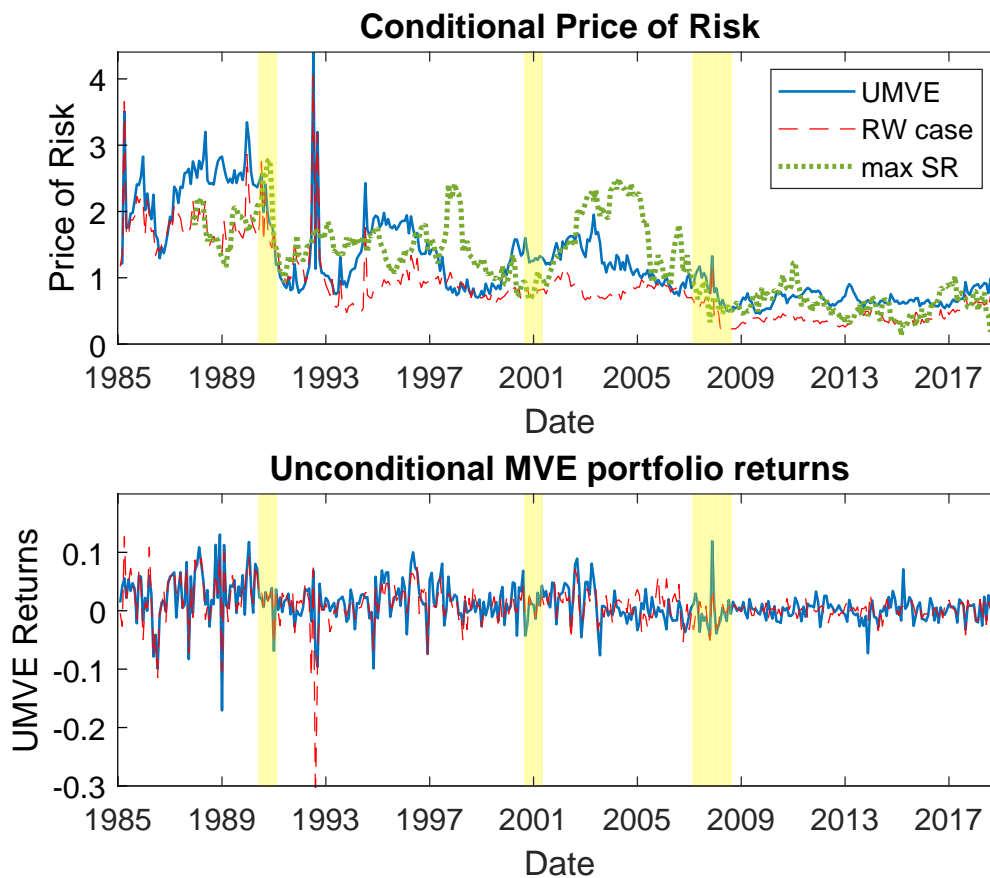
We deduce portfolio positions in the strategies that allow approximating the UMVE portfolio. We consider the Dollar strategy (we discard the Dollar Carry because it is conditionally perfectly correlated with the Dollar ), the sum of CS- and TS-Carry (labeled as Carry), the sum of CS- and TS-Momentum (labeled as Momentum), and CS-Value. We report 12-month moving average of the UMVE portfolio weights, which are expressed in decimal form. They have been scaled to match the (unconditional) standard deviation of the Dollar factor. The sample is monthly from January 1985 to May 2020.

**Figure 5**  
**Portfolios of strategies vs the UMVE-tracking portfolio**



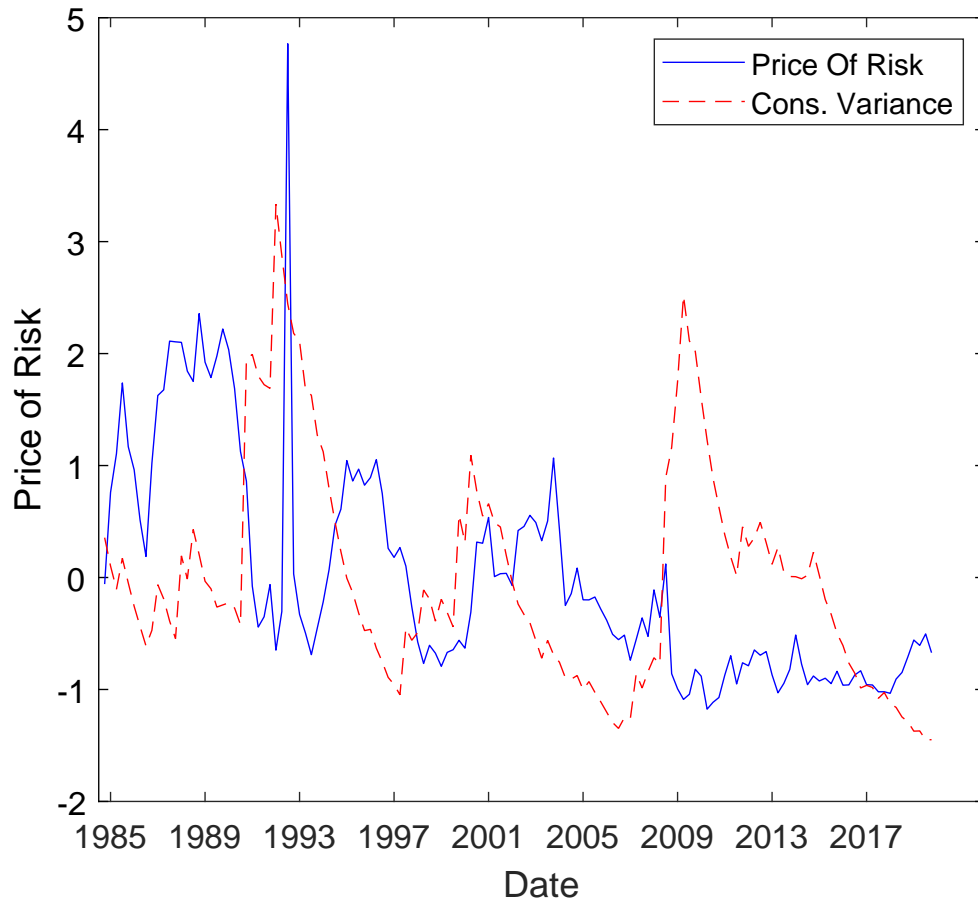
Panels (A)-(C) display differences between conditional Sharpe ratios of the UMVE proxy from Figure 4 and those of alternatives constructed from the same four strategies: 25% in each; weights inversely related to the conditional variance of each strategy (obtained from our conditional covariance matrix for individual depreciation rates); 50% in CS-Mom-12 and 50% in CS-Value. Panel (D) displays  $p$ -values of the Barillas-Shanken test of whether the UMVE proxy can be explained by an alternative portfolio (labeled UMVE proxy on LHS) or whether a fixed-weight portfolio can be explained by the UMVE proxy. The sample is monthly from January 1985 to May 2020.

**Figure 6**  
**Properties of the UMVE**



The top panel shows the annualized conditional price of risk as implied by the stochastic discount factor derived in the paper. The blue solid lines corresponds to the full model, whereas the dashed red lines shows the case where only the random walk assumption is used for estimating currency risk premiums. In addition, the green line depicts ex post maximal Sharpe Ratio, computed over the past 36 months, that serves as a quantitative benchmark. The lower panel shows the realized returns to the UMVE portfolio implied from this SDF. The color code is the same as for the top panel. The returns have been scaled to have the same standard deviation as the Dollar factor. The sample is monthly from January 1985 to May 2020.

**Figure 7**  
**Price of risk and consumption's variance**



The blue solid line is the conditional price of risk as implied by the stochastic discount factor derived in the paper. The dashed red line shows the conditional variance of quarterly, real per-capita nondurable+services log consumption growth, as estimated by a GARCH(1,1). Both series are standardized to be mean zero and unit variance to facilitate comparison. The sample is quarterly from end of December 1984 to end of April 2020.

Table 1: Predictive ability of conditional expectations and variance

Panel A		Currency returns	
	$R_{t,t+1}^{ei}$		$(R_{t,t+1}^{ei} - E_t R_{t,t+1}^{ei})^2$
$E_t(R_{t,t+1}^{ei})$	0.838	$V_t(R_{t,t+1}^{ei})$	0.931
s.e.	0.253	s.e.	0.068
$R_{adj}^2$	0.009	$R_{adj}^2$	0.053

Panel B		Strategy returns	
	$R_{t,t+1}^{ei}$		$(R_{t,t+1}^{ei} - E_t R_{t,t+1}^{ei})^2$
$E_t(R_{t,t+1}^{ei})$	0.791	$V_t(R_{t,t+1}^{ei})$	1.055
s.e.	0.237	s.e.	0.087
$R_{adj}^2$	0.003	$R_{adj}^2$	0.072

Panel C		Summary statistics of conditional means and variances		
	$E[E_t(R_{t,t+1}^{ei})]$	$E[V_t^{1/2}(R_{t,t+1}^{ei})]$	$V^{1/2}[E_t(R_{t,t+1}^{ei})]$	$V^{1/2}[V_t^{1/2}(R_{t,t+1}^{ei})]$
Currencies	0.021	0.107	0.044	0.048
Strategies	0.032	0.079	0.034	0.036

Panel D		UMVE portfolios		
Signals used	SR	alpha	$t$ -stat	$R_{adj}^2$
Carry+Mom+Val	1.075	n/a	n/a	n/a
Carry	0.871	0.0440	2.5369	0.515
Carry+Mom	0.871	0.0400	2.6327	0.586
Carry+Val	0.998	0.0181	2.1292	0.796

Panel A shows panel regressions of currency excess returns  $R_{t,t+1}^e$  on the conditional expected returns as estimated from our model,  $E_t R_{t,t+1}^e$ , and  $(R_{t,t+1}^e - E_t R_{t,t+1}^e)^2$  on the conditional variance as estimated by our model,  $V_t(R_{t,t+1}^e)$ . Panel B shows the same for the strategy returns. Standard errors are clustered on month. Panel C shows summary statistics for estimated conditional means and variances.  $E[E_t(R_{t,t+1}^e)]$  denotes the grand average risk premium,  $E[V_t^{1/2}(R_{t,t+1}^e)]$  is the grand average conditional standard deviation,  $V^{1/2}[E_t(R_{t,t+1}^e)]$  is the standard deviation of conditional expected returns, and  $V^{1/2}[V_t^{1/2}(R_{t,t+1}^e)]$  is the standard deviation of conditional standard deviations. Panel D shows the Sharpe ratio (SR) if the UMVE portfolio as a function of the signals used in its construction, along with the “alpha” of the full model UMVE portfolio regressed on alternative UMVE portfolios.

Table 2: Testing the UMVE

## Panel A Currency returns

Currency	SR	$E[R^e]$	$t$ -stat	$\alpha$	$t$ -stat	$R_{adj}^2$
AUD	0.241	2.83	1.44	0.73	0.34	0.025
CAD	0.099	0.74	0.59	-0.03	-0.02	0.007
CHF	0.149	1.68	0.89	3.22	1.56	0.014
EUR	0.145	1.52	0.86	1.83	0.95	-0.002
GBP	0.217	2.17	1.29	1.08	0.65	0.008
JPY	0.054	0.60	0.32	2.45	1.20	0.022
NOK	0.205	2.26	1.22	1.27	0.63	0.005
NZD	0.427	5.29	2.54	3.72	1.67	0.012
SEK	0.134	1.48	0.80	0.28	0.14	0.008

## Panel B Strategy returns

Strategy	SR	$E[R^e]$	$t$ -stat	$\alpha$	$t$ -stat	$R_{adj}^2$
Dollar	0.258	2.06	1.54	1.61	1.14	0.000
Dollar Carry	0.576	4.56	3.43	2.40	1.75	0.062
CS-Carry	0.469	4.14	2.79	0.43	0.28	0.151
TS-Carry	0.685	3.44	4.08	1.25	1.47	0.163
CS-Mom 1	0.179	1.46	1.06	0.61	0.36	0.007
CS-Mom 12	0.175	1.47	1.04	-1.22	-0.82	0.087
TS-Mom 1	0.440	2.73	2.62	1.45	1.30	0.034
TS-Mom 12	0.473	5.19	2.82	1.65	0.86	0.088
CS-Value	0.529	4.02	3.15	0.98	0.69	0.136

Panel A gives the annualized Sharpe ratio (SR), average return, and  $t$ -statistic of excess returns to each currency, along with its “alpha” and  $R^2$  with respect to the UMVE portfolio. The  $t$ -statistics are heteroskedasticity-adjusted. Panel B shows the same for the strategy returns. The sample is monthly from January 1985 to May 2020.

Table 3: Asset pricing tests

Model	UMVE		All strategies		Dollar + All Carry		
	SR	GRS	MHR	GRS hedged	GRS	MHR	GRS hedged
Optimal	1.075	0.470	0.277	0.475	0.467	0.568	0.434
Carry only	0.871	0.010	0.001	0.010	0.309	0.030	0.156
<i>V</i> -weighted	0.165	0.000	–	0.001	0.002	–	0.023
<i>E</i> -weighted	0.565	0.000	–	0.000	0.084	–	0.002

We contrast our main model of the UMVE, labeled as “Optimal,” with nested cases. The row labeled “Carry only” refers to conditional expectation of excess returns constructed using the interest rate differential only. The row labeled “*V*-weighted” corresponds to setting the conditional mean to a column of ones, while the row labeled “*E*-weighted” corresponds to setting the conditional covariance matrix to the identity matrix. The GRS  $p$ -value is for the joint test of all “alpha” equal to zero. The MHR  $p$ -value is for the conditional test of the model as implied by multihorizon returns. The horizons are 1, 3, 6, 12, 24, and 48 months. See the text for details. The “GRS hedged”  $p$ -value is for the joint test of all average hedged strategy returns equal to zero. For each strategy, we compute the conditional beta on the UMVE portfolio out-of-sample and thus hedge out the priced component, according to our model, in real time. This test is also an unconditional test of each model’s conditional implications. The sample is monthly from January 1985 to May 2020.

Table 4: Testing the UMVE: Alternative measurement currencies

Panel A Returns in AUD

Strategy	SR	$E[R^e]$	$t$ -stat	$\alpha$	$t$ -stat	$R_{adj}^2$
Dollar	0.177	1.42	1.05	1.11	0.78	-0.001
Dollar Carry	0.550	4.39	3.27	2.22	1.61	0.065
CS-Carry	0.397	3.58	2.36	-0.02	-0.02	0.145
TS-Carry	0.644	3.26	3.84	1.12	1.31	0.162
CS-Mom 1	0.185	1.55	1.10	0.58	0.33	0.010
CS-Mom 12	0.174	1.47	1.04	-1.23	-0.80	0.091
TS-Mom 1	0.475	5.28	2.83	1.70	0.88	0.093
TS-Mom 12	0.445	2.81	2.65	1.49	1.31	0.038
CS-Value	0.516	3.99	3.07	0.91	0.63	0.144
UMVE SR	1.042	GRS	$p$ -value	0.565		

Panel B Returns in JPY

Strategy	SR	$E[R^e]$	$t$ -stat	$\alpha$	$t$ -stat	$R_{adj}^2$
Dollar	0.204	1.62	1.22	1.17	0.83	0.000
Dollar-carry	0.556	4.37	3.31	2.26	1.65	0.058
CS-Carry	0.523	4.56	3.12	0.85	0.57	0.150
TS-Carry	0.703	3.52	4.19	1.33	1.56	0.158
CS-Mom 1	0.186	1.51	1.11	0.65	0.40	0.007
CS-Mom 12	0.179	1.49	1.06	-1.25	-0.85	0.088
TS-Mom 1	0.460	5.00	2.74	1.38	0.73	0.091
TS-Mom 12	0.433	2.64	2.58	1.42	1.28	0.031
CS-Value	0.547	4.11	3.26	1.10	0.79	0.132
UMVE SR	1.092	GRS	$p$ -value	0.455		

The table gives the annualized Sharpe ratio (SR), average return, and  $t$ -statistic of excess returns to each currency strategy, along with its “alpha” and  $R^2$  with respect to the UMVE portfolio. The  $t$ -statistics are heteroskedasticity-adjusted. Results for other measurement currency are similar and available upon request. The sample is monthly from January 1985 to May 2020.



Table 5: Testing the UMVE: Returns expressed in World Currency

Strategy	SR	$E[R^e]$	$t$ -stat	$\alpha$	$t$ -stat	$R_{adj}^2$
Dollar	0.177	1.42	1.05	1.06	0.75	-0.001
Dollar Carry	0.550	4.39	3.27	2.15	1.56	0.062
CS-Carry	0.397	3.58	2.36	0.35	0.23	0.144
TS-Carry	0.644	3.26	3.84	1.17	1.37	0.158
CS-Mom 1	0.185	1.55	1.10	0.64	0.38	0.008
CS-Mom 12	0.174	1.47	1.04	-1.26	-0.85	0.092
TS-Mom 1	0.475	5.28	2.83	1.52	0.79	0.092
TS-Mom 12	0.445	2.81	2.65	1.47	1.30	0.035
CS-Value	0.516	3.99	3.07	1.06	0.75	0.136
UMVE SR	1.067	GRS	$p$ -value	0.533		

The table gives the annualized Sharpe ratio (SR), average return, and  $t$ -statistic of excess returns to each currency strategy, along with its “alpha” and  $R^2$  with respect to the UMVE portfolio when returns are expressed in terms of a world exchange rate (constructed by equal-weighting depreciation rates across countries). The  $t$ -statistics are heteroskedasticity-adjusted. The sample is monthly from January 1985 to May 2020.

Table 6: Covariates

Panel A UMVE returns vs. observable factors

	Quarterly $\Delta c$	3-year $\Delta c$	HML	CMA
Beta	32.452	6.740	0.011	0.021
$t$ -stat	2.516	3.167	2.039	2.724
$R_{adj}^2$	0.031	0.073	0.006	0.013
N	141	131	425	425

Panel B UMVE, CS-Carry, its unpriced component and covariates

	CS-Carry			CS-Carry priced			CS-Carry unpriced		
	HKM	Equity RV	FX RV	HKM	Equity RV	FX RV	HKM	Equity RV	FX RV
Beta	0.111	-2.066	-6.023	0.009	-0.107	-0.836	0.102	-1.960	-5.187
( $t$ -stat)	3.482	-6.743	-6.628	1.066	-0.565	-1.494	3.458	-4.958	-4.652
R2adj	0.084	0.117	0.151	0.001	0.000	0.017	0.084	0.125	0.132
N	425	425	425	425	425	425	425	425	425

Panel C Price of risk vs. aggregate variance

	$\Delta$ Cons. Variance	$\Delta$ Equity RV	$\Delta$ FX RV
Beta	-22508.189	-1.478	-12.372
$t$ -stat	-3.928	-0.717	-2.806
$R_{adj}^2$	0.140	0.000	0.032
N	129	399	399

Panel A gives the results from regressing the UMVE portfolio returns on various factors in univariate regressions.  $\Delta c$  denote log real, per capita nondurable+services consumption growth. HML is the high-minus-low book-to-market factor from the Fama-French 5-factor model, whereas CMA is the Conservative minus Aggressive investment factor from the same. Panel B shows regressions of the CS Carry strategy, the priced component of CS carry, and the unpriced component of CS carry returns on the intermediary capital factor from [He, Kelly, and Manela \(2017\)](#),  $HKM$ , as well as shocks to realized variance in the equity and currency markets. Panel C displays a regression of the 3-year change in the conditional price of risk on the 3-year change in conditional consumption variance (obtained from a GARCH(1,1)), equity variance, and currency (FX) variance, respectively. We use 3-year changes instead of levels due to a downward drift in the price of risk over the sample. The  $t$ -statistics are heteroskedasticity-adjusted. The sample is monthly from January 1985 to May 2020.